MA26200: EXAM II **NAME:** ____________________

1. Which of the following sets are vector space?

- W_1= polynomials $P(t) = at^3 + bt^2 + ct + d$ satisfying $a - b + c = d$ with usual addition and scalar multiplication.
- W_2= degree ≤ 5 polynomials $P(t)$ satisfying $P(2) = 0$ with usual addition and scalar multiplication.
- W_3= vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ satisfying $x+y = 0$ with usual addition and scalar multiplication.
- W_4= all solutions of the linear system $Ax = 0$ for A an $m \times n$ matrix with usual addition and scalar multiplication.
- W_5= 3×3 upper triangular matrices with usual addition and scalar multiplication.

A. W_1, W_2, W_3, W_3

B. W_1, W_2, W_4, W_5

C. W_2, W_3, W_4, W_5

D. W_1, W_3, W_4, W_5

E. W_1, W_2, W_3, W_4, W_5

2. Consider the the vectors $v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ 1 \\ 7 \\ 3 \end{bmatrix}$, $v_3 = \begin{bmatrix} 5 \\ -3 \\ 9 \\ 1 \end{bmatrix}$, and $v_4 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 8 \end{bmatrix}$. The dimension of the space $\text{span}\{v_1, v_2, v_3, v_4\}$ is then equal to

A. 1

B. 2

C. 3

D. 4

E. 5

3. The subset $S = \{A \in M_3(\mathbb{R}^3) : A^T = -A\}$ of the space of 3×3 matrices with real elements is a vector space such that

A. dim$S$=2 and a basis for S is $\{ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \}$

B. dim$S$=2 and a basis for S is $\{ \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \}$

C. dim$S$=3 and a basis for S is $\{ \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix} \}$

D. dim$S$=3 and a basis for S is $\{ \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 2 & 0 \end{bmatrix} \}$

E. None of the above.

4. Write down the general solution of the following homogeneous linear equations:

(a) $2y'' + 3y' - 2y = 0$

(b) $y'' + 4y = 0$

(c) $y'' + 2y' + 4y = 0$

5. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be defined by $T(x) = Ax$ where $A = \begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 2 & -1 & 3 & 8 \end{bmatrix}$. Then a basis for Ker$(T)$ is given by the vectors

A. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \\ 8 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

6. Which of the following matrices are non-defective

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}?$$

A. A, B

B. A, C

C. B only

D. C only

E. A only

7. To find a particular solution of the inhomogeneous differential equation

$$(D-1)^2(D-2)(D^2+1)y = e^x + \cos x - 2\sin x$$

one can use the following solution

(a) $A_0 + A_1 \cos x + A_2 \sin x$

(b) $A_0 e^x + x(A_1 \cos x + A_2 \sin x)$

(c) $A_0 x e^x + x(A_1 \cos x + A_2 \sin x)$

(d) $A_0 x^2 e^x + x^2(A_1 \cos x + A_2 \sin x)$

(e) $A_0 x^2 e^x + x(A_1 \cos x + A_2 \sin x)$

8. The Wronskian of the functions $\{x, \sin x \cos x\}$ is equal to

A. $x(\cos x)(\sin x)$

B. $x(\cos^2 x - \sin^2 x)$

C. 0

D. x

E. $-x$

9. Let S be the subspace of $\mathbb{R}^3$ consisting of all vectors x of the form $x = (r+s, r-s, 2r+2s)$, r, s real. A basis for S is given by

A. $(1, 1, , 2)$, $(1, -1, 2)$

B. $(2, 0, 4)$, $(2, 0, 2)$

C. $(1, 1, 2)$, $(-1, 1, 2)$

D. $(2, 1, 1)$, $(2, -1, 1)$

E. $(1, 1, 2)$, $(2, -1, 1)$

10. Determine all values of k such that the vectors $(1, -1, 0)$, $(1, 2, 2)$, $(0, 3, k)$ are a basis for $\mathbb{R}^3$.

A. $k = 1$

B. $k = 2$

C. $k \neq 2$

D. $k \neq 1$

E. $k \neq 3$

11. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$.

What is $T(\begin{bmatrix} 1 \\ 2 \end{bmatrix})$?

A. $\begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 2 \\ 2 \\ 8 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$

E. $\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$