AN APPLICATION OF SECTIONS 1.4: COSMOLOGICAL SCIENCE

Example 2. According to one cosmological theory, there were equal amounts of the two uranium isotopes ^{235}U and ^{238}U at the creation of the universe in the big bang. At present there are 137.7 atoms of ^{238}U for each atom of ^{235}U. Using the half-lives 4.51×10^9 years for ^{238}U and 7.10×10^8 years for ^{235}U, calculate the age of the universe.

Solution. Let $N_8(t)$ and $N_5(t)$ be the numbers of ^{238}U and ^{235}U atoms, respectively, t billions of years after the big bang. Since both isotopes follow a radioactive decay model $x' = kx$, whose solution was seen in class to be $x(t) = x_0 e^{kt}$, we have

$$N_8 = N_0 e^{-kt},$$

and

$$N_5 = N_0 e^{-\ell t},$$

where N_0 is the initial number of atoms of each isotope, which is the same for both ^{238}U and ^{235}U by hypothesis. Notice however that the rates of decay, k and ℓ, differ for these isotopes. Their values are given by

$$N_8(4.51) = \frac{N_0}{2} = N_0 e^{-k \times 4.51} \Rightarrow k = \frac{\ln 2}{4.51},$$
$$N_5(0.71) = \frac{N_0}{2} = N_0 e^{-\ell \times 0.71} \Rightarrow \ell = \frac{\ln 2}{0.71}.$$

We know that for the value of t corresponding to "now" we have $\frac{N_8}{N_5} = 137.7$, hence

$$\frac{N_8}{N_5} = \frac{N_0 e^{-kt}}{N_0 e^{-\ell t}} = e^{(\ell - k)t} = e^{(\frac{\ln 2}{0.71} - \frac{\ln 2}{4.51})t} = 137.7.$$

Solving for t gives

$$t = \frac{\ln 137.7}{\frac{\ln 2}{0.71} - \frac{\ln 2}{4.51}} \approx 5.99.$$

According to this theory, therefore, the universe should be about 6 billion years old.