

EXAMPLES OF SECTIONS 7.1

Question 1. Transform the given DE into an equivalent system of first order DEs:

$$t^3 x''' - 2t^2 x'' + 3tx' + 5x = \ln t.$$

Question 2. Write the system of question 1 in matrix form.

SOLUTIONS.

1. Let $x_1 = x$, $x_2 = x'_1 = x'$, $x_3 = x'_2 = x''$, hence

$$x'_3 = x''' = \frac{-5x - 3tx' + 2t^2 x'' + \ln t}{t^3},$$

or

$$x'_3 = -\frac{5}{t^3}x_1 - \frac{3}{t^2}x_2 + \frac{2}{t}x_3 + \frac{\ln t}{t^3}.$$

Therefore

$$\begin{cases} x'_1 = & x_2 \\ x'_2 = & x_3 \\ x'_3 = & -\frac{5}{t^3}x_1 - \frac{3}{t^2}x_2 + \frac{2}{t}x_3 + \frac{\ln t}{t^3} \end{cases}$$

2. We readily see that if

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

and

$$\vec{b} = \begin{bmatrix} 0 \\ 0 \\ \frac{\ln t}{t^3} \end{bmatrix}$$

then

$$\vec{x}' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{5}{t^3} & -\frac{3}{t^2} & \frac{2}{t} \end{bmatrix} \vec{x} + \vec{b}.$$