EXERCISES OF CHAPTER 4

Question 1. Which of the following vectors in $\mathbb{R}^3$ is a linear combination of

$$v_1 = \begin{bmatrix}4 & 2 & -3\end{bmatrix}, \quad v_2 = \begin{bmatrix}2 & 1 & -2\end{bmatrix}, \quad v_3 = \begin{bmatrix}-2 & -1 & 4\end{bmatrix}?$$

A. $\begin{bmatrix}1 & 0 & 0\end{bmatrix}$

B. $\begin{bmatrix}1 & 1 & 1\end{bmatrix}$

C. $\begin{bmatrix}-2 & 2 & 3\end{bmatrix}$

D. $\begin{bmatrix}6 & 3 & 7\end{bmatrix}$

E. None of the above.

Question 2. Let P_3 be the set of all polynomials of degree 3 or less. Which of the following subsets are subspaces of P_3? (Here all subsets are assumed to have the usual standard addition and scalar multiplication)

(i) all polynomials $p(x)$ such that $p(1) \neq 0$.

(ii) all polynomials $p(x)$ such that $p(x) = p(-x)$.

(iii) all polynomials $p(x)$ such that $p(1) = p(0)$.

A. (i) and (ii)

B. (ii) only.

C. (ii) and (iii)

D. (i) and (iii)

E. All of the above are vector spaces.

Question 3. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ -1 & -2 & -3 & -1 \end{bmatrix}.$$

What is the dimension of null space of A?

Question 4. What is the dimension of the subspace of R^4 spanned by $\{(1,2,3,4),(4,3,2,1),(2,0,0,2),(2,4,4,2)\}$?

Question 5. For which values of the constant k, do the vectors $(2,1,3k,4)$, $(0,k-1,4,-8)$, $(0,0,2,1)$, $(0,0,k,4)$ form a basis for $\mathbb{R}^4$?

Question 6. Find the value of k such that $(5, 6k, -2, 2)$ is in the span of $\{(0, 2, 2, 1), (-1, 0, 2, 1), (2, 2, 0, 3)\}$?

Question 7. Determine all values of k so that $\{k - kx^2, 3 + kx, 2 + x + kx^2\}$ is a basis for P_2, the vector space of all polynomials of degree ≤ 2.

Question 8. What is the dimension of the vector space of all 4×4 skew-symmetric matrices with real entries?

Question 9. Find all value(s) of k, such that the rowspace of A is $\mathbb{R}^3$, where

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & k \\ 2 & 0 & 1 \\ 0 & 1 & k \end{bmatrix}.$$