EXERCISES OF MATRICES OPERATIONS

Throughout, we assume that the dimensions of the matrices in this note make sense.

Question 1. Which of the following statements must be true?

(1) If $A^2 = A$, then A must be either the identity matrix or the zero matrix.

(2) If A is a 2×2 matrix and $|A| = 4$, then $|2A| = 8$.

(3) If $A^T = -A$, then $|A| = 0$.

(4) If $A^2 = I$, then $A = I$ or $A = -I$.

(5) If $A^2 = 0$, then $A = 0$.

(6) If $AB = 0$, then $A = 0$ or $B = 0$.

(7) If A is an $n \times n$ matrix and $A^n = 0$, then $A = 0$.

(8) If A is symmetric, then A^T is symmetric.

(9) If A is symmetric, then $-A$ is skew-symmetric.

(10) If A, B is symmetric, then AB is symmetric.

(11) If A, B is symmetric, then $A + B$ is symmetric.

(12) If $AC = BC$, then $A = B$.

(13) If $CA = CB$, then $A = B$.

(14) If C is invertible and $AC = BC$, then $A = B$.

(15) If $AB = I_n$, then so is BA.

(16) If A, B are both $n \times n$ matrices and $AB = I_n$, then so is BA.

(17) The nullity of A is the same as the nullity of A^T.

(18) Both A, B are invertible, then so is $A + B$.

(19) Both A, B are invertible, then so is AB.

(20) If A, B are both $n \times n$ matrices and AB is invertible, then both A, B are invertible.

(21) Both A, B are singular, then so is $A + B$.

(22) Both A, B are singular, then so is AB.

(23) If A, B are both $n \times n$ matrices and AB is singular, then both A, B are singular.

(24) If A, B are both $n \times n$ matrices and AB is singular, then A is singular or B is singular.

(25) A is diagonalizable, then A is non-singular.

(26) A is symmetric, then A is non-singular.

(27) If all the eigenvalues of A are 1, then A is similar to the identity matrix.

(28) If all the eigenvalues of A are 1, then A is non-singular.

(29) If A is invertible, then A^2 is invertible.

(30) If A is invertible, then AA^T is invertible.

(31) If A is invertible, then A^T is invertible.

Question 2. If A is row equivalent to B, then which of the following statements must be true?

(1) If λ is an eigenvalue of A, then λ is also an eigenvalue of B.

(2) A can be obtained from B by a finite step of elementary row operations.

(3) $AX = 0$ and $BX = 0$ has the same solutions.

(4) $AX = b$ and $BX = b$ has the same solutions for any b.

(5) $\text{rank}(A) = \text{rank}(B)$.

(6) A, B have the same reduced row echelon form.

(7) AC and BC are row equivalent for any C.