MA26500: EXAM I **NAME:** ____________________

1. A is an $m \times n$ matrix and b is an $m \times 1$ vector. The equation $AX = b$ has infinitely many solutions. Consider the following statements:

(i) $m \leq n$

(ii) $n \leq m$

(iii) $\text{rank}(A) = n$

(iv) $\text{rank}(A) < n$

(v) $\det(A) = 0$

Which must be true?

A. only (i) and (v)

B. only (iv)

C. only (v)

D. only (iii) and (v)

E. None of the above.

2. Only one of the following is NOT always true. Which one is it?

A. The product of two 3×3 diagonal matrices is a diagonal matrix.

B. For any two $n \times n$ matrices A, B, $(A+B)^2 = A^2 + AB + BA + B^2$.

C. For any two matrices A, B, if $AB = 0$ then either $A = 0$ or $B = 0$.

D. Product of two 3×3 upper triangular matrices is an upper triangular matrix.

E. The transpose of a symmetric matrix is symmetric.

3. If $A = \begin{bmatrix} 0 & 2 & -2 \\ 2 & 3 & 0 \\ 1 & 5 & -3 \end{bmatrix}$. What is the **sum** of the entries in the third row of A^{-1}?

A. $-\frac{5}{2}$

B. $\frac{5}{2}$

C. $\frac{3}{2}$

D. 1

E. 5

4. Which of the following are true?

(i) An $n \times n$ elementary matrix is always nonsingular.

(ii) If A is nonsingular and diagonal, then A^{-1} is also nonsingular and diagonal.

(iii) An $n \times n$ matrix is nonsingular if and only if its reduced row echelon form is I_n.

(iv) If A is symmetric, then $\text{adj}A$ is also symmetric.

A. only (i) and (ii)

B. (i), (ii), (iii)

C. (i), (ii), (iv)

D. (ii), (iii), (iv)

E. (i), (ii), (iii), (iv)

5. Let A be an invertible matrix with the inverse

$$A^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}.$$

Which of the following statement is NOT always true?

(i) For arbitrary 2×2 matrices B and C. If $AB = AC$, then $B = C$.

(ii) A^T is invertible.

(iii) For arbitrary 2×2 matrices B and C. If $BA = CA$, then $B = C$.

(iv) rank(A)=2

(v) A is symmetric.

A. (i) and (iii)

B. (ii) and (v)

C. (i), (ii), (iii) and (v)

D. (i), (iii) and (v)

E. None of the above.

6. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & 1 & 0 \\ -1 & -4 & 1 & -1 \\ 1 & 2 & 0 & -4 \end{bmatrix}$$

via **the row echelon form method**.

7. For two $n \times n$ matrices A and B, how many of the following statements are true.

(a) $\det(AB) = \det(A)\det(B)$

(b) $\det(A) = \det(A^T)$

(c) For $k \neq 0$, $\det(kA) = k\det(A)$.

(d) $\det(A^{-1}) = \det(A)^{-1}$

(e) If $A = PBP^{-1}$ for an invertible matrix P, then $\det(A) = \det(B)$.

A. 1

B. 2

C. 3

D. 4

E. 5

8. For what α, the system of linear equations

$$\begin{cases} 2x + 5y + (3\alpha)z + 4w &= 0 \\ (\alpha - 1)y + 4z - 3w &= 0 \\ 2z + w &= 0 \\ (\alpha)z + 4w &= 0 \end{cases}$$

has non-trivial solutions?

A. $\alpha = 0, 2$

B. $\alpha = 1, 5$

C. $\alpha = -1, -5$

D. $\alpha = 1, 8$

E. $\alpha = 0, 1$

9. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

and

$$AX = \begin{bmatrix} -1 \\ 2 \\ -4 \end{bmatrix}$$

What is x_2?

A. 11

B. 20/3

C. 5/4

D. 1

E. 0

10. Which of the following sets are vector space?

- W_1= polynomials $P(t) = at^3 + bt^2 + ct + d$ satisfying $a - b + c = d$ with usual addition and scalar multiplication.
- W_2= degree ≤ 5 polynomials $P(t)$ satisfying $P(2) = 0$ with usual addition and scalar multiplication.
- W_3= vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ satisfying $x+y = 0$ with usual addition and scalar multiplication.
- W_4= all solutions of the linear system $Ax = 0$ for A an $m \times n$ matrix with usual addition and scalar multiplication.
- W_5= 3×3 upper triangular matrices with usual addition and scalar multiplication.

A. W_1, W_2, W_3, W_3

B. W_1, W_2, W_4, W_5

C. W_2, W_3, W_4, W_5

D. W_1, W_3, W_4, W_5

E. W_1, W_2, W_3, W_4, W_5

11. Determine which one of the following expressions is the general solution to the homogeneous system of equations

$$\begin{cases} x_1 + 5x_2 + 4x_3 + 3x_4 + 2x_5 & = 0 \\ x_1 + 6x_2 + 6x_3 + 6x_4 + 6x_5 & = 0 \\ x_1 + 7x_2 + 8x_3 + 10x_4 + 12x_5 & = 0 \\ x_1 + 6x_2 + 6x_3 + 7x_4 + 8x_5 & = 0 \end{cases}$$

A. $s\begin{bmatrix} 1 \\ 0 \\ -6 \\ 0 \\ 6 \end{bmatrix}$

B. $s\begin{bmatrix} 6 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -6 \\ 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$

C. $s\begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 6 \end{bmatrix}$

D. $s\begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 0 \\ -4 \\ -1 \\ 7 \\ 0 \end{bmatrix}$

E. $s\begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 0 \\ -4 \\ -1 \\ 7 \\ 0 \end{bmatrix} + r\begin{bmatrix} 1 \\ -3 \\ -1 \\ 5 \\ 1 \end{bmatrix}$