MA26500: EXAM II **March 31, 2016**

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NAME: ____________________ **PUID:**____________________

Section Number: ____________________ **Class Time:**____________________

(1) No calculators are allowed.

(2) No portable electronic devices.

(3) There are 10 problems. Each problem is worth 10 points.

(4) The score is accumulative and the maximum is 100.

1. The subspace of $\mathbb{R}^3$ spanned by $\{(1,-1,0),(1,-2,1),(1,4,1),(1,-6,1)\}$ has dimension

A. 0

B. 1

C. 2

D. 3

E. 4

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -2 & 4 & -6 \\ 0 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 6 & -4 \end{bmatrix}$$

So the first 3 vectors are L.I.

2. If the vector $\begin{bmatrix} 2 \\ 1 \\ a \end{bmatrix}$ is in the column space of $\begin{bmatrix} 3 & 6 \\ 2 & 5 \\ 4 & 7 \end{bmatrix}$, then $a =$

A. 3

B. 2

C. 1

D. 0

E. There is no such value of a.

$$\left[\begin{array}{cc|c} 3 & 6 & 2 \\ 2 & 5 & 1 \\ 4 & 7 & a \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 2/3 \\ 0 & 1 & -1/3 \\ 0 & -3 & a-2 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 2 & 2/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & a-3 \end{array}\right]$$

This system is consistent if and only if $a = 3$

3. Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors $(1,0,0,0)$, $(k, \overset{0}{\cancel{2}}, \overset{2}{\cancel{4}}, \overset{8}{\cancel{10}})$, $(k,1,1,1)$, $(-k,1,k,k^2)$. Determine all values of the constant k such that the dimension of $W^{\perp}$ is 0.

A. no value of k

B. $k \neq 1, 3$

C. $k \neq 1, 2$

D. $k = 1, 3$

E. $k = 1, 2$

$$\dim W^{\perp} = 0 \iff W = \mathbb{R}^4$$

$$\iff 0 \neq \begin{vmatrix} 1 & k & k & -k \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & k \\ 0 & 8 & 1 & k^2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 1 & k \\ 8 & 1 & k^2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 2 & 1 & k-1 \\ 8 & 1 & k^2-1 \end{vmatrix}$$

$$= - \begin{vmatrix} 2 & k-1 \\ 8 & k^2-1 \end{vmatrix} = -2k^2 + 8k - 6 = -2(k-3)(k-1)$$

4. Which of the following sets of vectors in $M_{2\times 2}$ are linearly independent?

(i) $\left\{\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\right\}$

(ii) $\left\{\begin{bmatrix}1 & 0\\ 1 & 2\end{bmatrix}, \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\right\}$

(iii) $\left\{\begin{bmatrix}1 & 2\\ 1 & -4\end{bmatrix}, \begin{bmatrix}0 & 1\\ -1 & 4\end{bmatrix}, \begin{bmatrix}2 & 5\\ 1 & -4\end{bmatrix}\right\}$

(iv) $\left\{\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}, \begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}, \begin{bmatrix}0 & 2\\ 1 & 1\end{bmatrix}\right\}$

A. (i), (ii), (iii)

B. (ii), (iii), (iv)

C. (i), (iii), (iv)

D. (i), (ii), (iv)

E. None of the above.

(ii) $\text{rank}\begin{bmatrix}1 & 1\\ 0 & 2\\ 1 & 0\\ 2 & 1\end{bmatrix} = 2$

(iii) $A = \begin{bmatrix}1 & 0 & 2\\ 2 & 1 & 5\\ 1 & -1 & 1\\ -4 & 4 & -4\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 1\\ 0 & -1 & -1\\ 0 & 4 & 4\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

$\text{rank } A = 2$

(iv) $A = \begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 2\\ 2 & 1 & 1\\ 1 & 4 & 1\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 0\\ 0 & -4 & 2\\ 0 & -3 & 1\\ 0 & 2 & 1\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 0\\ 0 & 2 & 1\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$

$\text{rank } A = 3$

5. A is a real 3×3 matrix and $N(A) = \{0\}$. Consider the following statements:

(i) $|A| \neq 0$

(ii) $\text{rank} A = 3$

(iii) the column space of A is $\mathbb{R}^3$

(iv) A^{-1} exists

which of these statements **must** be true

A. only (i) and (iv)

B. only (ii) and (iii)

C. only (i) and (ii)

D. None of them have to be true.

E. All of them have to be true.

$N(A) = 0 \Leftrightarrow A$ invertible.

6. Let v_1, v_2, v_3, v_4 be an **orthogonal basis** of $\mathbb{R}^4$ with the standard inner product. Let $W = \text{span}\{v_1, v_2\}$ and let u be a vector in the orthogonal complement of W. Then which of the following need NOT be true?

A. $v_1,\ v_2,\ v_3 + u,\ v_4 + u$ is a basis of $\mathbb{R}^4$.

B. $v_1 + u,\ v_2 + u,\ v_3,\ v_4$ is a basis of $\mathbb{R}^4$.

C. $v_1 + v_3$ is orthogonal to $v_2 + v_4$.

D. $v_1 + v_2$ is orthogonal to $v_3 + v_4 + u$.

E. $\text{proj}_W(v_1 + v_2 + u) = \text{proj}_W(v_1 + v_2 + v_3 + v_4)$.

A. A might not true. Consider $u = -V_3$. Then $u \perp W$

but $\{V_1\ V_2,\ 0,\ V_4 - V_3\}$ is not a basis

B. This set is a spanning set $\Rightarrow$ a basis

C. $(V_1 + V_3,\ V_2 + V_4) = (V_1, V_2) + (V_1, V_4) + (V_3, V_2) + (V_3, V_4) = 0$

D. The same as in C.

E. $\text{Proj}_W(V_1 + V_2 + u) = V_1 + V_2 = \text{Proj}_W(V_1 + V_2 + V_3 + V_4)$

7. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation for which

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-2\end{bmatrix}, \quad T\left(\begin{bmatrix}-1\\1\end{bmatrix}\right) = \begin{bmatrix}2\\3\end{bmatrix}.$$

Then $T\left(\begin{bmatrix}1\\-5\end{bmatrix}\right) =$

A. $\begin{bmatrix}-8\\-5\end{bmatrix}$

B. $\begin{bmatrix}8\\5\end{bmatrix}$

C. $\begin{bmatrix}-8\\5\end{bmatrix}$

D. $\begin{bmatrix}8\\-5\end{bmatrix}$

E. $\begin{bmatrix}-5\\-8\end{bmatrix}$

$$\begin{bmatrix}1\\-5\end{bmatrix} = c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}-1\\1\end{bmatrix}$$

$$\Rightarrow \quad c_1 = -2 \qquad c_2 = -3$$

$$T\left(\begin{matrix}1\\-5\end{matrix}\right) = -2\begin{bmatrix}1\\-2\end{bmatrix} - 3\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}-8\\-5\end{bmatrix}$$

8. Suppose that $W =$ the plane $x + 2y - 3z = 0$. Which of the following is a basis for $W^{\perp}$?

A. $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

D. $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & -3 \end{bmatrix}$$

$$W = N(A)$$

$$W^{\perp} = \text{rowspace}(A) = S\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

9. Let W denote the vector space spanned by the vectors

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

and let $v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$. Find the distance from v to W.

A. 0

B. 1

C. 2

D. $\sqrt{2}$

E. 4

$$(v, u_1) = 0 = (v, u_2)$$

$$\Rightarrow \quad v \perp W$$

$$\Rightarrow \quad \mathrm{Proj}_W v = 0$$

$\Rightarrow$ distance of v to W

$$= \| v - \mathrm{Proj}_W v \| = \| v \| = 2.$$

10. Which of the following set is NOT a basis of $\mathbb{R}^3$?

A. $\{(1,0,0),(0,2,0),(0,0,3)\}$

B. $\{(1,-1,0),(2,-1,0),(3,0,-1)\}$

C. $\{(1,-2,0),(0,2,-3),(-1,0,3)\}$

D. $\{(1,1,0),(2,1,0),(3,0,1)\}$

E. $\{(0,2,0),(1,2,3),(0,0,3)\}$

C. $\det\begin{bmatrix} 1 & 0 & -1 \\ -2 & 2 & 0 \\ 0 & -3 & 3 \end{bmatrix} = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 0 & -3 & 3 \end{vmatrix} = \begin{vmatrix} 2 & -2 \\ -3 & 3 \end{vmatrix} = 0$

1 D

2 A

3 B

4 D

5 E

6 A

7 A

8 C

9 C

10 C