EXERCISES OF MATRICES OPERATIONS - SOLUTIONS

Throughout, we assume that the dimensions of the matrices in this note make sense.

Question 1. Which of the following statements must be true?

(1) F. Counterexample: $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(2) F.

(3) F. Counterexample: $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(4) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(5) F. Counterexample: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(6) F. Counterexample: the same as above.

(7) F. Counterexample: A is the $n \times n$ with 1 on the $(n-1)$ entries to the right of the main diagonal and zero elsewhere.

(8) T.

(9) F. $-A$ is still symmetric.

(10) F.

(11) T.

(12) F. Counterexample: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(13) F.

(14) T.

(15) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(16) T.

(17) F. Counterexample: A is a 3×5 matrix with rank 2.

(18) F. Counterexample: $A = I_n$ and $B = -I_n$.

(19) T.

(20) T. Because: $0 \neq |AB| = |A||B| \Longrightarrow |A| \neq 0, |B| \neq 0$.

(21) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(22) T.

(23) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(24) T.

(25) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(26) F. Counterexample: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(27) F. Counterexample: $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(28) T.

(29) T.

(30) T.

(31) T.

Question 2. If A is row equivalent to B, then which of the following statements must be true?

(1) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(2) T.

(3) T.

(4) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(5) T.

(6) T.

(7) T.