EXAMPLES OF SECTIONS 3.3

Question 1. Suppose that A is an $n \times n$ matrix. Show that if there exists a matrix B such that $BA = I_n$, then $AB = I_n$.

SOLUTIONS.

1. First note that $BA = I_n$ implies that B is also an $n \times n$ matrix. Taking determinant on both sides of $BA = I_n$ yields

$$|B||A| = |BA| = |I_n| = 1 \Longrightarrow |A| \neq 0.$$

Recall that in class, we proved that this implies A is invertible. Thus there exists A^{-1} such that $A^{-1}A = AA^{-1} = I_n$. Now applying a similar argument we saw in Section 1.5, we have

$$A^{-1} = I_n A^{-1} = BAA^{-1} = BI_n = B.$$

Hence $AB = AA^{-1} = I_n$.