EXERCISES OF CHAPTER 7

Question 1. A is an $n \times n$ matrix. Which of the following must be true?

a If all the eigenvalues of A are 1, then A is similar to the diagonal matrix.

b If all the eigenvalues of A are 1 and A is symmetric, then A is similar to the diagonal matrix.

c If all the eigenvalues of A are distinct, then the corresponding eigenvectors form a basis for $\mathbb{R}^n$.

d If all the eigenvalues of A are distinct, then the corresponding eigenvectors form an orthonormal basis for $\mathbb{R}^n$.

e If all the eigenvalues of A are distinct and A is symmetric, then the corresponding eigenvectors is an orthogonal set.

Question 2. Suppose A is a symmetric 2×2 matrix with two distinct eigenvalues λ_1, λ_2. Which of the following statements MUST be true?

(i) A is similar to $\begin{bmatrix} \lambda_2 & 0 \\ 0 & \lambda_1 \end{bmatrix}$.

(ii) A is diagonalizable.

(iii) If v_1 is an eigenvector with respect to λ_1 and v_2 is an eigenvector with respect to λ_2, then $\{v_1, v_2\}$ is a basis of $\mathbb{R}^2$.

(iii) If v_1 is an eigenvector with respect to λ_1 and v_2 is an eigenvector with respect to λ_2, then $\{v_1, v_2\}$ is an orthonormal of $\mathbb{R}^2$.

Question 3. Find a matrix P such that $P^{-1}AP$ is a diagonal matrix, where

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}.$$