

$$1. \quad k^3 + 2k^2 - 1 = 0 \quad k = -1 \text{ is a root}$$

$$\text{By long division, } 0 = (k+1)(k^2+k-1) \quad k = 1, \frac{-1 \pm \sqrt{5}}{2}$$

$$2. \quad \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$S \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$3. \quad \begin{vmatrix} 1 & 0 & 2 \\ 2 & k & 4 \\ 1 & 1 & k \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & k & 0 \\ 1 & 1 & k-2 \end{vmatrix} = k(k-2) = 0 \Rightarrow k=0 \text{ or } k=2$$

$$4. \quad A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & 0 & -1 & 1 \\ 1 & 3 & 2 & 4 \\ 1 & 1 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 3 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \left(\text{You can also put these} \right. \\ \left. \text{vectors in rows.} \right)$$

$$\text{rank } A = 2 \Rightarrow \dim W = 2 \Rightarrow \dim W^\perp = 2$$

$$5. \quad v = 2u_1 + u_2 \Rightarrow \text{distance} = 0$$

$$6. \quad \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right) = [x_1 \ x_2 \ x_3] \begin{bmatrix} 3 & 3 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 3x_1y_1 + 3x_1y_2 + 3x_2y_1 + 4x_2y_2 + 4x_3y_3$$

$$\text{So } (e_1, e_2) = 3 \quad \|e_1\|^2 = 3 \quad \|e_2\|^2 = 4$$

$$\cos \theta = \frac{3}{\sqrt{3} \cdot 2} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$$