EXERCISES OF MATRICES OPERATIONS

Throughout, we assume that the dimensions of the matrices in this note make sense.

**Question 1.** Which of the following statements must be true?

(1) If $A^2 = A$, then $A$ must be either the identity matrix or the zero matrix.

(2) If $A$ is a $2 \times 2$ matrix and $|A| = 4$, then $|2A| = 8$.

(3) If $A^T = -A$, then $|A| = 0$.

(4) If $A^2 = I$, then $A = I$ or $A = -I$.

(5) If $A^2 = 0$, then $A = 0$.

(6) If $AB = 0$, then $A = 0$ or $B = 0$.

(7) If $A$ is an $n \times n$ matrix and $A^n = 0$, then $A = 0$.

(8) If $A$ is symmetric, then $A^T$ is symmetric.

(9) If $A$ is symmetric, then $-A$ is skew-symmetric.

(10) If $A, B$ is symmetric, then $AB$ is symmetric.
(11) If $A, B$ is symmetric, then $A + B$ is symmetric.

(12) If $AC = BC$, then $A = B$.

(13) If $CA = CB$, then $A = B$.

(14) If $C$ is invertible and $AC = BC$, then $A = B$.

(15) If $AB = I_n$, then so is $BA$.

(16) If $A, B$ are both $n \times n$ matrices and $AB = I_n$, then so is $BA$.

(17) The nullity of $A$ is the same as the nullity of $A^T$.

(18) Both $A, B$ are invertible, then so is $A + B$.

(19) Both $A, B$ are invertible, then so is $AB$.

(20) If $A, B$ are both $n \times n$ matrices and $AB$ is invertible, then both $A, B$ are invertible.

(21) Both $A, B$ are singular, then so is $A + B$.

(22) Both $A, B$ are singular, then so is $AB$.

(23) If $A, B$ are both $n \times n$ matrices and $AB$ is singular, then both $A, B$ are singular.
(24) If $A, B$ are both $n \times n$ matrices and $AB$ is singular, then $A$ is singular or $B$ is singular.

(25) $A$ is diagonalizable, then $A$ is non-singular.

(26) $A$ is symmetric, then $A$ is non-singular.

(27) If all the eigenvalues of $A$ are 1, then $A$ is similar to the identity matrix.

(28) If all the eigenvalues of $A$ are 1, then $A$ is non-singular.

(29) If $A$ is invertible, then $A^2$ is invertible.

(30) If $A$ is invertible, then $AA^T$ is invertible.

(31) If $A$ is invertible, then $A^T$ is invertible.

**Question 2.** If $A$ is row equivalent to $B$, then which of the following statements must be true?

1. If $\lambda$ is an eigenvalue of $A$, then $\lambda$ is also an eigenvalue of $B$.

2. $A$ can be obtained from $B$ by a finite step of elementary row operations.

3. $AX = 0$ and $BX = 0$ has the same solutions.
(4) $AX = b$ and $BX = b$ has the same solutions for any $b$.

(5) $\text{rank}(A) = \text{rank}(B)$.

(6) $A, B$ have the same reduced row echelon form.

(7) $AC$ and $BC$ are row equivalent for any $C$. 