PROOF FOR SKEW-SYMMETRIC MATRICES

Example 1. *Suppose* $A_{n\times n} = [a_{ij}]$ *is a skew symmetric matrix. Prove that* $a_{ii} = 0$ *for* $i = 1, 2, \cdots, n$ *and* $a_{ij} = -a_{ji}$ *for* $i \neq j$.

Solution. We denote the (i, j)-th entry of A^T by a_{ij}^T. We have shown in class that

$$a_{ij}^T = a_{ji}.$$

Together with $A^T = -A$, this yields

$$a_{ij} = a_{ji}^T = -a_{ji}$$

for all $i, j = 1, 2, \cdots, n$. The second equality comes from the definition of $A^T = -A$. When $i = j$, in particular, we have

$$a_{ii} = -a_{ii} \Longrightarrow a_{ii} = 0.$$

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