(19) Find the eigenvalues and associated eigenvectors of the following matrix:

$$A = \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$$

$|\lambda I - A| = \begin{vmatrix} \lambda - 2 & -2 \\ -5 & \lambda + 1 \end{vmatrix}$

$= \lambda^2 - \lambda - 12 = (\lambda - 4)(\lambda + 3)$

$\lambda_1 = 4$

$\begin{bmatrix} 2 & -2 \\ -5 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = -3$

$\begin{bmatrix} -5 & -2 \\ -5 & -2 \end{bmatrix} \sim \begin{bmatrix} 5 & 2 \\ 0 & 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

A. $\lambda_1 = -3$, $\lambda_2 = 4$, $\mathbf{x}_1 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

B. $\lambda_1 = -3$, $\lambda_2 = 4$, $\mathbf{x}_1 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

C. $\lambda_1 = -3$, $\lambda_2 = -4$, $\mathbf{x}_1 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

D. $\lambda_1 = 3$, $\lambda_2 = 4$, $\mathbf{x}_1 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

E. $\lambda_1 = 3$, $\lambda_2 = -4$, $\mathbf{x}_1 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(20) Which of the following matrices is **not** diagonalizable?

A. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

B, C, E two distinct eigenvalues

A. $\lambda = 0$ is an eigenvalue of multiplicity 2

$0 \cdot I - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ has nullity 2. So $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is diagonalizable.

D $\lambda = 1$ is an eigenvalue of multiplicity 2

$1I - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

So it is not diagonalizable

(17) What is the characteristic polynomial of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-1 & -2 \\ 0 & -2 & \lambda-1 \end{vmatrix}$$

$$= (\lambda-1)\begin{vmatrix} \lambda-1 & -2 \\ -2 & \lambda-1 \end{vmatrix} = (\lambda-1)(\lambda^2-2\lambda-3)$$

$$= (\lambda-1)(\lambda+1)(\lambda-3)$$

A. $(\lambda-1)(\lambda+2)$
B. $(\lambda-1)(\lambda-1)(\lambda+1)$
C. $(\lambda-1)(\lambda+1)(\lambda-3)$
D. $(\lambda-1)(\lambda+1)(\lambda+3)$
E. $(\lambda+1)(\lambda+1)(\lambda-3)$

(18) Let $A = PDP^{-1}$ where $P = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, $D = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}$ and $P^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$.

Then A^3 equals

$$A^3 = PD^3P^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 27 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -29 & 56 \\ -28 & 55 \end{bmatrix}$$

A. $\begin{pmatrix} -29 & 56 \\ -28 & 55 \end{pmatrix}$

B. $\begin{pmatrix} -29 & 56 \\ -28 & 57 \end{pmatrix}$

C. $\begin{pmatrix} -29 & 55 \\ -28 & 54 \end{pmatrix}$

D. $\begin{pmatrix} -24 & 56 \\ -25 & 57 \end{pmatrix}$

E. $\begin{pmatrix} -24 & 56 \\ -25 & 55 \end{pmatrix}$