MA26500: Quiz 1 **March 6, 2017**

NAME: ____________________ **Class Time:**____________________

(1) No calculators are allowed.

(2) No portable electronic devices.

(3) There are 2 problems. Each problem is worth 6 points.

1. Let $A = \begin{bmatrix} 1 & 3 & -1 & 7 \\ 5 & 1 & 5 & 7 \\ 7 & 0 & 8 & 7 \end{bmatrix}$.

(i) Find a basis for the column space of A. (2 pt)

(ii) Find a basis for the row space of A. (1 pt)

(iii) What is the nullity of A? (1 pt)

(iv) Is the vector $\begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$ in the column space of A? (2 pt)

(i)
$$\begin{bmatrix} 1 & 3 & -1 & 7 \\ 5 & 1 & 5 & 7 \\ 7 & 0 & 8 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -1 & 7 \\ 0 & -14 & 10 & -28 \\ 0 & -21 & 15 & -42 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 7 \\ 0 & 7 & -5 & 14 \\ 0 & 7 & -5 & 14 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -1 & 7 \\ 0 & 7 & -5 & 14 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\left\{ \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a basis for colspace(A)

(ii) $\left\{ \begin{matrix} [1 \ \ 3 \ \ -1 \ \ 7] \\ [0 \ \ 7 \ \ -5 \ \ 14] \end{matrix} \right\}$ is a basis for rowspace(A)

(iii) nullity of $A = 4 - \text{rank } A = 2$

(iv)
$$\left[\begin{array}{cccc|c} 1 & 3 & -1 & 7 & 0 \\ 5 & 1 & 5 & 7 & -4 \\ 7 & 0 & 8 & 7 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 3 & -1 & 7 & 0 \\ 0 & 7 & -5 & 14 & 2 \\ 0 & 7 & -5 & 14 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & -1 & 7 & 0 \\ 0 & 7 & -5 & 14 & 2 \\ 0 & 0 & 0 & 0 & -3 \end{array}\right]$$ no solution

$\begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}$ is NOT in colspace(A)

2. Suppose that $S = \{at^3 + bt^2 + ct + d : a + b = c - d,\ 2a + 3b + c = 0\}$ is a subspace of $\mathbb{P}_3$? ($\mathbb{P}_3$ is the space of all polynomials of degree no more than 3)

(i). Find a basis for S. (3 pt)

(ii). Is $\{t^3 - t^2 + t + 1, t^3 + t^2 + 2t\}$ a basis for S? (3 pt)

(i) $\mathbb{R}^4 \supseteq S = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{cases} a + b - c + d = 0 \\ 2a + 3b + c = 0 \end{cases} \right\}$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 3 & -2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \end{array}\right] \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4s & -3t \\ -3s & +2t \\ s & \\ & t \end{bmatrix}$$

$$= s \begin{bmatrix} 4 \\ -3 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix}.$$

So $\{4t^3 - 3t^2 + t,\ -3t^3 + 2t^2 + 1\}$ is a basis for S.

(ii) $t^3 + t^2 + 2t \notin S$. So this is NOT a basis.

$t^3 + t^2 + t + 1 \in S$.