

EXAMPLES OF SECTIONS 1.8-2

Question 2. Solve

$$(x^4 - 2t^3x)dt + (t^4 - 2tx^3)dx = 0.$$

Solutions.

2. We divide both sides of the equation by t^4 and transform it into

$$\frac{dx}{dt} = \frac{(x/t)^4 - 2x/t}{1 - 2(x/t)^3}.$$

This is a homogeneous equation and thus we can make the substitution $V(t) = x/t$.

$$tV' + V = \frac{V^4 - 2V}{1 - 2V^3} \implies tV' = 3\frac{V^4 - V}{1 - 2V^3} \implies \int \frac{1 - 2V^3}{V^4 - V} dV = \int \frac{3}{t} dt + C.$$

Using partial fraction and noticing that $V^4 - V = V(V - 1)(V^2 + V + 1)$, we have

$$\frac{1 - 2V^3}{V^4 - V} = \frac{1}{V} - \frac{1}{3} \frac{1}{V - 1} - \frac{1}{3} \frac{2V + 1}{V^2 + V + 1}.$$

This yields

$$\ln(|V| \sqrt[3]{V - 1} \sqrt[3]{V^2 + V + 1}) = -9 \ln |t| + C.$$