

EXAMPLES OF SECTIONS 2.4

Question 1. Find the reduce echelon form of the augmented coefficient matrix of the following linear system:

$$\begin{cases} 2x_1 - x_2 + 3x_3 - x_4 = 3 \\ 3x_1 + 2x_2 + x_3 - 5x_4 = -6 \\ x_1 - 2x_2 + 3x_3 + x_4 = 6. \end{cases}$$

SOLUTIONS.

1. The augmented matrix of the system is

$$\left[\begin{array}{cccc|c} 2 & -1 & 3 & -1 & 3 \\ 3 & 2 & 1 & -5 & -6 \\ 1 & -2 & 3 & 1 & 6 \end{array} \right]$$

Then

$$\begin{aligned} & \left[\begin{array}{cccc|c} 2 & -1 & 3 & -1 & 3 \\ 3 & 2 & 1 & -5 & -6 \\ 1 & -2 & 3 & 1 & 6 \end{array} \right] \xrightarrow{P_{13}} \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 6 \\ 3 & 2 & 1 & -5 & -6 \\ 2 & -1 & 3 & -1 & 3 \end{array} \right] \\ & \xrightarrow{A_{12}(-3)} \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 6 \\ 0 & 8 & -8 & -8 & -24 \\ 2 & -1 & 3 & -1 & 3 \end{array} \right] \xrightarrow{M_2(\frac{1}{8})} \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 6 \\ 0 & 1 & -1 & -1 & -3 \\ 2 & -1 & 3 & -1 & 3 \end{array} \right] \\ & \xrightarrow{A_{13}(-2)} \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 6 \\ 0 & 1 & -1 & -1 & -3 \\ 0 & 3 & -3 & -3 & -9 \end{array} \right] \xrightarrow{A_{23}(-3)} \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 6 \\ 0 & 1 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{aligned}$$

$$\widetilde{A_{21}(2)} \begin{bmatrix} 1 & 0 & 1 & -1 & \vdots & 0 \\ 0 & 1 & -1 & -1 & \vdots & -3 \\ 0 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$