EXAMPLES OF SECTIONS 4.4

Question 1. Find solution vectors $\vec{u}$ and $\vec{v}$ such that the solution space is the set of all linear combinations of the form $s\vec{u} + t\vec{v}$:

$$\begin{cases} x_1 - 4x_2 - 3x_3 - 7x_4 = 0 \\ 2x_1 - x_2 + x_3 + 7x_4 = 0 \\ x_1 + 2x_2 + 3x_3 + 11x_4 = 0 \end{cases}$$

SOLUTIONS.

1. The augmented matrix of the system is

$$\begin{bmatrix} 1 & -4 & -3 & -7 & \vdots & 0 \\ 2 & -1 & 1 & 7 & \vdots & 0 \\ 1 & 2 & 3 & 11 & \vdots & 0 \end{bmatrix}.$$

Applying Gauss-Jordan elimination we find

$$\begin{bmatrix} 1 & 0 & 1 & 5 & \vdots & 0 \\ 0 & 1 & 1 & 3 & \vdots & 0 \\ 0 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix}.$$

Therefore x_3 and x_4 are free variables. Denoting by $x_3 = s$, $x_4 = t$, we can then write

$$x_1 = -s - 5t,$$
$$x_2 = -s - 3t.$$

Therefore solutions $\vec{x} = (x_1, x_2, x_3, x_4)$ can be written as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -s-5t \\ -s-3t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix} = s\vec{u} + t\vec{v},$$

where

$$\vec{u} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}.$$