1. Find a particular solution of

$$y'' - 3y' - 4y = 2e^{-t}.$$

A. $Y(t) = 2e^{-t}$

B. $Y(t) = 2t$

C. $Y(t) = te^{-t}$

D. $Y(t) = \frac{2}{5}e^{-t}$

E. $\boxed{Y(t) = -\frac{2}{5}te^{-t}}$

2. The form of a particular solution to the equation

$$y'' + 2y' + 2y = te^t \cos t$$

is:

A. $(At + B)e^t \cos t$

B. $t(At + B)e^t \cos t$

C. $Ate^t \cos t + Bte^t \sin t$

D. $\boxed{(At + B)e^t \cos t + (Ct + D)e^t \sin t}$

E. $t(At + B)e^t \cos t + t(Ct + D)e^t \sin t$

3. The homogeneous differential equation $y'' - \frac{4}{t}\, y' + \frac{6}{t^2}\, y = 0$ $(t > 0)$ has two solutions given by $y_1(t) = t^2$ and $y_2(t) = t^3$. Using the method of Variation of Parameters, find the general solution of the nonhomogeneous equation $y'' - \frac{4}{t}\, y' + \frac{6}{t^2}\, y = t$.

A. $y = C_1t^2 + C_2t^3 + \ln t$

B. $y = C_1t^2 + C_2t^3 + t \ln t$

C. $y = C_1t^2 + C_2t^3 + t^2 \ln t$

D. $\boxed{y = C_1t^2 + C_2t^3 + t^3 \ln t}$

E. $y = C_1t^2 + C_2t^3 + t^4 \ln t$

4. A mass weighing 2 lb stretches a spring 6 in. If the mass is pulled down an additional 3 in. and then released, and if there is no damping, determine the position u of the mass at any time t.

A. $u(t) = \sin(2t)$

B. $u(t) = \cos(4t)$

C. $u(t) = \sin(2t) + \cos(4t)$

D. $u(t) = \frac{1}{4}\sin(\sqrt{2}t)$

E. $\boxed{u(t) = \frac{1}{4}\cos(8t)}$

5. A certain vibrating system satisfies the equation $u'' + \gamma u' + u = 0$. Find the value of the damping coefficient γ for which the quasi period of the damped motion is 50% greater than the period of the corresponding undamped motion.

A. $\gamma = \sqrt{2}$

B. $\gamma = 2$

C. $\gamma = 0$

D. $\gamma = 3$

E. $\boxed{\gamma = \sqrt{3}}$

6. Which of the following set of functions is linearly independent?

A. $f_1(t) = 1, \quad f_2(t) = t, \quad f_3(t) = t - 2$

B. $f_1(t) = 1, \quad f_2(t) = t^2 + 1, \quad f_3(t) = t^2 - 1$

C. $\boxed{f_1(t) = 1, \quad f_2(t) = 2t - 3, \quad f_3(t) = t^2 + t + 1}$

D. $f_1(t) = 2t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = 2t^2 - t, \quad f_4(t) = t^2 + t + 1$

E. $f_1(t) = 2t - 3, \quad f_2(t) = 2t^2 + 1, \quad f_3(t) = 3t^2 + 1$

7. Find the general solution of

$$y^{(4)} - 5y'' + 4y = 0$$

A. $y(t) = c_1 e^t + c_2 e^{-t}$

B. $\boxed{y(t) = c_1 e^t + c_2 e^{-t} + c_3 e^{2t} + c_4 e^{-2t}}$

C. $y(t) = c_1 e^{2t} + c_2 e^{-2t}$

D. $y(t) = c_1 e^t + c_2 e^{2t}$

E. $y(t) = c_1 e^{-t} + c_2 e^{-2t}$

8. Find the general solution of

$$y''' + 8y = 0.$$

A. $y(t) = c_1 e^{-t} \sin \sqrt{3}t + c_2 e^{-t} \cos \sqrt{3}t + c_3 e^{-2t}$

B. $y(t) = c_1 \sin t + c_2 \cos t + c_3 e^{-2t}$

C. $\boxed{y(t) = c_1 e^t \sin \sqrt{3}t + c_2 e^t \cos \sqrt{3}t + c_3 e^{-2t}}$

D. $y(t) = c_1 e^{-t} \sin \sqrt{3}t + c_2 e^{-t} \cos \sqrt{3}t + c_3 e^{2t}$

E. $y(t) = c_1 e^{\sqrt{2}t} \sin \sqrt{2}t + c_2 e^{\sqrt{2}t} \cos \sqrt{2}t + c_3 e^{-2t}$

9. Find a particular solution of the equation

$$y''' - 4y' = t + 3\cos t + e^{-2t}$$

A. $y(t) = -t + \frac{1}{2}\cos t + e^{-2t}$

B. $y(t) = t^2 + \frac{1}{2}\cos t - e^{-2t}$

C. $y(t) = -t + \frac{1}{2}\sin t + \frac{1}{8}te^{-2t}$

D. $\boxed{y(t) = -\frac{1}{8}t^2 - \frac{3}{5}\sin t + \frac{1}{8}te^{-2t}}$

E. $y(t) = t^2 + \frac{1}{2}\cos t - te^{-2t}$

10. Let y be the solution of the initial value problem

$$y'' + y = 2\sin t, \quad y(0) = 2, \quad y'(0) = 1.$$

The Laplace transform of y is:

A. $Y(s) = \dfrac{2s}{s^2+1}$

B. $Y(s) = \dfrac{5/3}{s^2+1}$

C. $Y(s) = \dfrac{2/3}{s^2+4}$

D. $\boxed{Y(s) = \dfrac{2s}{s^2+1} + \dfrac{5/3}{s^2+1} - \dfrac{2/3}{s^2+4}}$

E. $Y(s) = \dfrac{2s}{s^2+1} + \dfrac{5/3}{s^2+1}$

11. The inverse Laplace transform of

$$\frac{s^3+2}{s^4+s^2}$$

is

A. $t^2 + 2\cos t + \sin t$

B. $\cos t + 2\sin t$

C. $2\sin t - \cos t$

D. $\boxed{2t + \cos t - 2\sin t}$

E. $t + 1 + 2\sin t$

12. The Laplace transform of the function

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2t, & t \ge 1 \end{cases}$$

is

A. $s^{-2}(1 + e^{-s})$

B. $\boxed{s^{-2} + e^{-s}(s^{-2} + s^{-1})}$

C. $s^{-2}(1 + e^{-2s})$

D. $s^{-2}(1 + \frac{1}{4}e^{-s})$

E. $e^{-s}(s^{-1} + \frac{1}{4}s^{-2})$