Journal of Computational Physics 250 (2013) 555–564

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

A GPU parallelized spectral method for elliptic equations in rectangular domains ☆

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ARTICLE INFO

Article history:
Received 19 November 2012
Received in revised form 20 May 2013
Accepted 21 May 2013
Available online 3 June 2013

Keywords:
Diagonalization
Elliptic equation
Spectral-collocation method
CUDA
GPU

ABSTRACT

We design and implement a polynomial-based spectral method on graphic processing units (GPUs). The key to success lies in the seamless integration of the matrix diagonalization technique and the new generation CUDA tools. The method is applicable to elliptic equations in rectangular domains with general boundary condition. We show remarkable speedups of up to 15 times in the 2-D case and more than 35 times in the 3-D case.

1. Introduction

General-purpose computing on graphics processing units (GPGPU) has drawn much attention recently from the scientific computing community. The new list of top 500 supercomputers shows that more than 10% of them are now powered by NVIDIA Tesla GPUs (cf. [4]). Thanks to the high core density and the wide vector width SIMD architecture, using GPGPU can yield performance that is very hard for a conventional CPU to achieve, especially for high and regular throughput workloads (cf. [3]). Therefore, researchers in computational sciences are more and more interested in exploring efficient parallel strategies on GPUs.

There exist many successful GPU implementations of numerical methods for partial differential equations, to name a few, fast multipole methods [10], nodal discontinuous Galerkin methods [12], finite difference methods [6], finite element methods [24,14,13], Fourier spectral methods [9,5], etc. However, to the best of the authors' knowledge, not much effort, if not at all, are devoted to implementing, on GPUs, non-Fourier based spectral methods for problems with non-periodic boundary conditions. The aim of this paper is to exploit how to efficiently implement spectral methods with GPGPU. In particular, we shall design and implement an efficient matrix diagonalization method with spectral discretizations on the GPU, for solving elliptic and parabolic type equations with non-periodic boundary conditions on a d-dimensional rectangular domain ($d = 2, 3$).

It is well-known that, for separable elliptic equations, one can use the so-called matrix diagonalization technique (cf. [16,11,18]) to solve the linear systems from a spectral discretization in $O(N^{d+1})$ operations, where N is the number of points in each dimension. For time-dependent problems, these solvers are called repeatedly. Fortunately, the core part of this

☆ The research of Feng Chen is partly supported by OSD/AFOSR Grant FA9550-09-1-0613 and AFOSR Grant FA9550-12-1-0463. The research of Jie Shen is partly supported by NSF Grant DMS-1217066 and AFOSR Grant FA9550-11-1-0328.
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http://dx.doi.org/10.1016/j.jcp.2013.05.031