

# **PL Homeomorphisms and Related Topics**

*Titles and Abstracts*

Purdue University

August 10-14, 2026

## Mini-courses

**Takashi Tsuboi** (*The University of Tokyo / RIKEN / Tohoku University*)

**Title:** *The Godbillon-Vey invariant and the group of PL homeomorphisms*

**Abstract:** In the mini-course, I will talk about an attempt to characterize the Godbillon-Vey invariant for codimension-1 foliations of 3-manifolds in terms of cobordisms, as an application of the study of PL homeomorphisms. This attempt was done about 35 years ago, and it was really fortunate at that time that there appeared the work on the classifying space for PL foliations by Peter Greenberg and the works on the GV invariants by Hurder-Katok and Ghys-Sergiescu, which led us to think about the domain of definition of the GV invariant. Thus in the mini-course, I will begin by explaining what the domain of definition of the GV invariant should be. Then we will consider the following statement:  $GV(M^3, F) = 0$  if and only if  $(M^3, F)$  is cobordant to  $(M^3_{\infty}, F_{\infty})$ , which is the limit of null-cobordant foliations  $(M^3_i, F_i)$ . If we would like  $(M^3_i, F_i)$  to be PL foliated  $S^1$ -products to use Greenberg's results, we will see what is necessary to prove. First,  $(M^3_{\infty}, F_{\infty})$  can be taken as a foliated  $S^1$ -product over  $\Sigma_2$  (the closed surface of genus 2). Then the holonomy over generators of  $\pi_1(\Sigma_2)$  is approximated by PL homeomorphisms in a reasonable sense. The product of commutators of approximations is a PL homeomorphism near the identity and it should be written as a fixed number of commutators of PL homeomorphisms near the identity. Finally, if  $GV(M^3_i, F_i) = 0$  then  $(M^3_i, F_i)$  is null-cobordant in a reasonable class of foliations. I will explain these in the mini-course.

**Yash Lodha** (*Purdue University*)

**Title:** *Groups of piecewise projective homeomorphisms*

**Abstract:** I will present a gentle introduction to groups of piecewise projective homeomorphisms of the circle and the real line. These groups exhibit a remarkable combination of algebraic, analytic and topological properties. This will cover some work of Peter Greenberg, Nicolas Monod, and the work of the speaker with Justin Moore.

**Sam Nariman** (*Purdue University*)

**Title:** *Greenberg's Perspective on Haefliger Spaces of PL Homeomorphisms*

**Abstract:** Mather-Thurston theory shows that the homology of a diffeomorphism group is closely related to the homotopy type of the classifying space of Haefliger structures. I will begin by explaining this relationship to motivate why Haefliger spaces matter. Although the homotopy type of Haefliger spaces for smooth foliations remains mysterious, Greenberg developed an inductive approach for studying the PL case. I will sketch Greenberg's ideas, starting with codimension one and then describing the inductive process, which becomes increasingly intricate in higher codimension. In the final lecture, I will discuss some applications to the homological properties of groups of PL homeomorphisms.

## Talks

### **Hyun Kyu Kim** (*KIAS*)

**Title:** *Central extensions of Thompson group  $T$  and quantum Teichmüller theory*

**Abstract:** Quantum Teichmüller theory for a finite type surface yields a projective representation of the mapping class group. For the universal case it yields a projective representation of an asymptotically rigid mapping class group of the unit disc, which is isomorphic to the Thompson group  $T$ . This in turn gives a central extension of  $T$  by the group of integers  $\mathbb{Z}$ , which is also obtained by relative abelianization of the asymptotically rigid mapping class group of an infinitely punctured unit disc. We review known results, and some open questions, especially regarding the Greenberg-Sergiescu extension and the discrete Godbillon-Vey class.

### **Thomas Koberda** (*University of Virginia*)

**Title:** *PL homeomorphisms and a first order Rubin theory*

**Abstract:** I will discuss joint work with J. de la Nuez Gonzalez in which we prove that many groups of homeomorphisms of manifolds, including groups of PL homeomorphisms, have very rich first order theories. I will discuss an analogue of Rubin's theorem for these groups: if  $G$  is a sufficiently complicated group of PL homeomorphisms of a manifold  $M$ , and  $H$  is elementarily equivalent to  $G$ , then the only manifold on which  $H$  can act with (a certain technically defined) sufficient complexity is  $M$  itself.

### **Martín Gilabert Vio** (*Institut Camille Jordan in Lyon*)

**Title:** *Groups with classifiable actions on the line*

**Abstract:** We investigate and motivate the class  $C$  of countable groups  $G$  such that the conjugacy relation between minimal actions of  $G$  on  $\mathbb{R}$  admits a Borel transversal. We show several closure properties of  $C$  under group-theoretic constructions, and that all finitely generated groups of piecewise affine homeomorphisms of  $\mathbb{R}$  belong to  $C$ . The purpose of the talk is to define the previous terms and to outline some ideas that appear in proofs. Based on work with Joaquín Brum and Nicolás Matte Bon.

### **Cary Malkiewich** (*Binghamton University*)

**Title:** *Higher scissors congruence*

**Abstract:** Scissors congruence is the study of polytopes, up to the relation of cutting into finitely many pieces and rearranging the pieces. This intersects Greenberg's work in two ways: he studied a smooth version of scissors congruence himself in 1983, and he studied polygons under  $SL_2(\mathbb{Z})$ -actions in 1993. In the 2010s, Zakharevich defined a "higher" version of scissors congruence, where we do not just ask whether two polytopes are scissors congruent, but also how many scissors congruences there are from one polytope to another. Zakharevich's definition is a form of algebraic K-theory, which is famously difficult to compute, but I will discuss a surprising result that makes the computation of the higher K-groups possible, and a homological stability result that allows us to relate these calculations to the group of cut-and-paste transformations of any polytope. In particular, we get the homology of the group of interval exchange transformations, and a new proof of Szymik and Wahl's theorem that Thompson's group  $V$  is acyclic. Much of this talk is based on joint work with Anna-Marie Bohmann, Teena Gerhardt, Mona Merling, and Inna Zakharevich, and also with Alexander Kupers, Ezekiel Lemann, Jeremy Miller, and Robin Sroka.

### **Louisa Liles** (*Ohio State University*)

**Title:** *Thompson's Groups and Knot Theory*

**Abstract:** In 1965, R. Thompson defined the groups  $F \subset T \subset V$ , which have since been studied in the context of group theory, homotopy theory, and knot theory. In 2014 Vaughan Jones defined a surjective map from  $F$  to the set of isotopy classes of links in the 3-sphere, giving the first example of a finitely generated group that produces all possible link types. Jones's map gave rise to new oriented subgroups  $F$ -vector and  $T$ -vector, corresponding to oriented links. We will introduce Jones's map and then discuss some recent extensions to tangles, braids, links in the thickened annulus, links in more general thickened surfaces, and their invariants. This talk includes joint work with Micah Chrisman, Slava Krushkal, Yangxiao Luo, Melody Molander, and Susanna Terron.

**Mee Seong Im** (*Johns Hopkins University*)

**Title:** *Sah-Arnoux-Fathi invariants, algebraic K-theory and foam cobordisms*

**Abstract:** I will introduce scissor congruence and algebraic K-theory and how they are related to cobordism groups of foams in various dimensions. The former is joint with Mikhail Khovanov while the latter is joint with David Gepner, Mikhail Khovanov, and Nitu Kitchloo.

**Matt Zaremsky** (*University at Albany/SUNY*)

**Title:** *The state of the art of the Boone-Higman conjecture, and the role of Thompson groups*

**Abstract:** In the 1970's, Boone and Higman conjectured that every finitely generated group with solvable word problem embeds in a finitely presented simple group. In this talk I will discuss recent progress on this conjecture, and highlight the important role that Thompson-like groups play. In particular, in recent joint work with Francesco Fournier-Facio and Xiaolei Wu, we prove that whenever a group satisfies the Boone-Higman conjecture, it also satisfies a related embedding conjecture entirely in the world of Thompson-like groups. This talk will also touch on joint work with Jim Belk and James Hyde.

**Sander Kupers** (*University of Toronto*)

**Title:** *Cocycle representatives for Borel regulator classes.*

**Abstract:** I will present some aspects of joint work with Daniil Rudenko and Ismael Sierra, in which we find new cocycle representatives for the Borel regulator classes in the cohomology of the general linear group of the complex numbers. These are of a polylogarithmic nature, generalising previous formulas of Bloch and Goncharov.

**Michele Triestino** (*Institut de Mathématiques de Marseille*)

**Title:** *Thompson's group  $T$  and its subgroups*

**Abstract:** The subgroup structure of Thompson's groups has been investigated in several works, but the picture is still very partial. I will discuss several problems and some preliminary results, notably for the case of  $T$ .

**Richard Kenyon** (*Yale*)

**Title:** *Tilings of convex polygons by convex polygons*

**Abstract:** We study tilings of polygons  $R$  with arbitrary convex polygonal tiles. Such tilings come in continuous families obtained by moving tile edges parallel to themselves (keeping edge directions fixed). We study how the tile shapes and areas change in these families. In particular we show that if  $R$  is convex, the tile shapes can be arbitrarily prescribed (up to homothety). We also show that the tile areas and tile "orientations" determine the tiling.

**Karim Adiprasito** (*IMJ-PRG/CNRS*)

**Title:** *The Hodge theory of lattice polytopes*

**Abstract:** I will describe algebraic geometry and Hodge/Lefschetz theorems for lattice polytopes, using topology of semigroup rings in positive characteristic. I conclude with a proof of the Ohsugi-Hibi conjecture.