SUPPLEMENT 1

- (1) Is the set of all functions $f : \mathbf{Q} \to \mathbf{Q}$ countable? Prove your answer.
- (2) Let $A = \{1, 1/2, 1/3, ...\} \subset \mathbf{R}$. Are all points of A (or some of them) interior (the surrounding metric space is \mathbf{R})? Are they limit points? Isolated ones? Is 0 a limit point? Is A open? Is it closed?
- (3) This is an exotic example. Let X be the set consisting of finitely many distinct points in \mathbf{R}^n . Prove that X is a metric space (cite a theorem). Describe all open and all closed subsets of X.
- (4) Prove that the empty set and the whole **R** are both open and closed in **R**. Prove that they are the only two subsets with that property.
- (5) What is the answer of the second question above if we replace \mathbf{R} by \mathbf{Q} ?