# HW #5, A SUPPLEMENTARY QUESTION

This is a form of the Phragmén-Lindelöf Principle, which we saw in a different form already. It is essentially a maximum principle for certain unbounded domains *under an additional a priori estimate*.

(a) Let $f(z)$ be an analytic function in the sector $D$ between two rays making an angle $\pi/\alpha$ at the vertex, and continuous on $\bar{D}$. Assume that

$$|f(z)| \le M \tag{1}$$

on the rays with some $M > 0$, and

$$|f(z)| \le Ce^{|z|^\beta} \quad \text{in } D, \tag{2}$$

with some $\beta < \alpha$. Prove that (1) holds in $D$.

(b) For $0 < \alpha$ fixed, find a function $f$ which satisfies the assumptions above except for the exponential estimate (2), for which the conclusion fails.