CONFERENCE ON INVERSE PROBLEMS IN HONOR OF GUNTHER UHLMANN

UC IRVINE, JUNE 18–22, 2012

ORGANIZED BY

Stephen McDowall
Plamen Stefanov
Andras Vasy
Lihan Wang
Maciej Zworski

Contents

Plenary Talks

1 Guillaume Bal, *Inverse Problems with Internal Functionals*

Wednesday, 9:45–10:30.

ABSTRACT: The reconstruction of coefficients in partial differential equations (PDE) from knowledge of functionals of the PDE solutions and the coefficients is an inverse problem that finds applications in several medical imaging modalities. This talk reviews several mathematical results obtained recently for such inverse problems, including injectivity and stability properties and the derivation of explicit inversion procedures.

2 Gang Bao, *Inverse Scattering Problems: Theory, Computation, and Applications*

Friday, 9:00–9:45.

ABSTRACT:

3 Liliana Borcea, *Imaging in random media*

Friday, 9:45–10:30.

ABSTRACT: Coherent interferometric imaging is based on the backpropagation of local space-time cross correlations of echoes measured at an array. It was introduced in order to improve standard imaging when the medium between the array and the object to be imaged is inhomogeneous and unknown. I will present an analysis of the resolution and statistical fluctuations of images when the ambient medium is random and scattering. The analysis shows that coherent interferometry can be understood as a denoising process that enhances the signal-to-noise ratio of images, but also reduces the resolution. I will also explain how coherent interferometric imaging can be done experimentally. Specifically, I will show that the imaging function is equivalent to a windowed beamformer energy function, that is, a quadratic function that involves only time gating and time delaying signals in emission and in reception.

4 Margaret Cheney, *Radar Imaging*

Friday, 11:00–11:45.

ABSTRACT: This talk will deal with recent work on radar imaging

5 David Colton, *Transmission Eigenvalues and Inverse Scattering Theory*

Thursday, 11:00–11:45.

ABSTRACT: The transmission eigenvalue problem is a non-self adjoint eigenvalue problem that arises in the inverse scattering problem for non-absorbing media. The interest lies in the fact that transmission eigenvalues can be determined from the scattering data and provide information about the index of refraction of the media. In this talk we will give a brief survey of this problem including the existence of eigenvalues,monotonicity properties of the first real eigenvalue and the inverse spectral problem for spherically stratified media.Numerical examples will be given together with a list of open problems.

6 Allan Greenleaf, *Can there be a general theory of Fourier integral operators?*

Thursday, 9:45–10:30.

ABSTRACT: Fourier integrals are ubiquitous in inverse problems, integral geometry and spectral theory. The transverse/clean intersection calculus is often sufficient to describe the composition of FIOs, but in many other cases the degeneracy of the underlying canonical relations puts the composition outside the classical theory. In fact, the composition of degenerate FIOs forces one to consider Fourier integral distributions associated not to smooth Lagrangian manifolds, but rather to Lagrangians with singularities. I will discuss how such objects have occurred in Gunther's work, some more recent results, and pose some problems.

7 Colin Guillarmou, *Semiclassical limits of plane wave and Eisenstein functions*

Tuesday, 9:45–10:30.

ABSTRACT: We study semiclassical measures of distorted plane waves functions on manifolds with Euclidean or hyperbolic ends, when the trapped set has Liouville measure 0. This is joint work with S. Dyatlov (UC Berkeley).

8 David Isaacson, *Problems that led me to Gunther Uhlmann*

Monday, 11:45–12:30.

ABSTRACT: I describe some problems that led me to meet Gunther Uhlmann. I was interested in understanding heart disease. This led me to inverse problems in electro and magneto cardiography which in turn led to electrical impedance imaging. I will describe these and if time permits more problems that caused me to cross paths with Gunther Uhlmann. Movies and pictures will be shown illustrating some of the problems that arose in trying to build systems to image heart and lung function.

9 Carlos Kenig, *Limiting Carleman weights and inverse problems*

Wednesday, 9:00–9:45.

ABSTRACT: We will discuss the notion of limiting Carleman weights and their use in local inverse problems and in anisotropic ones. This is based on joint works with Dos Santos Ferreira, Salo, Sjostrand and Uhlmann.

10 Matti Lassas, *Cloaked wave amplifiers via transformation optics*

Tuesday, 9:00–9:45.

ABSTRACT: The advent of transformation optics and metamaterials has made possible devices producing extreme effects on wave propagation. Here we give theoretical designs for devices, Schrodinger hats, acting as invisible concentrators of waves. These exist for any wave phenomenon modeled by either the Helmholtz or Schrodinger equations, e.g., polarized waves in electromagnetism, pressure waves in acoustics and matter waves in quantum mechanics, and occupy one part of a parameter space continuum of wave-manipulating structures which also contains standard transformation optics based cloaks, resonant cloaks and cloaked sensors. For electromagnetic and acoustic Schrodinger hats, the resulting centralized wave is a localized excitation. In quantum mechanics one can use Schrodinger hats to reserve probability mass of particles and create probabilistic illusions.

We discuss possible solid state implementations. The presented results are obtained in collaboration with Allan Greenleaf, Yaroslav Kurylev, Ulf Leonhardt, and Gunther Uhlmann.

11 Rafe Mazzeo, *Singular Monge-Ampere equations in geometry*

Monday, 9:45–10:30.

ABSTRACT: I will report on some recent progress concerning the complex Monge Ampere equation, centered on the problem of existence of Kähler-Einstein metrics bent along a possibly singular divisor. After describing several geometric and analytic motivations, I will present a new existence theorem, the proof of which relies on a surprising blend of fully nonlinear and geometric microlocal techniques.

12 Joyce McLaughlin, *Biomechanical Imaging of Tissue from Frequency Content Data*

Wednesday, 2:00–2:45.

ABSTRACT:

13 Richard Melrose, *Adiabatic limits and eigenvalues*

Friday, 11:45–12:30.

ABSTRACT:

14 Lassi Paivarinta, *Non-scattering energies and transmission eigenvalues*

Wednesday, 11:00–11:45.

ABSTRACT:

15 Mikko Salo, *Geodesic ray transforms and tensor tomography*

Monday, 11:00–11:45.

ABSTRACT: The standard X-ray transform, where one integrates functions over straight lines, is a well-studied object and forms the basis of medical imaging techniques such as CT and PET. This transform has useful generalizations involving other families of curves, weight factors, and integration of tensor fields. These more general transforms come up in seismic imaging (inverse kinematic / boundary rigidity problem), in medical imaging (SPECT and ultrasound), and in the mathematical analysis of other inverse problems such as the Calderon and Gel'fand problems.

In this talk we discuss recent progress in the analysis of ray transforms, focusing on the case where one integrates over a family of geodesics. The talk is based on joint works with Gabriel Paternain (Cambridge) and Gunther Uhlmann (Washington / UC Irvine)

16 Fadil Santosa, *Resonances of Low-loss Resonators*

Wednesday, 11:45–12:30.

ABSTRACT: This work is motivated by the desire to develop a method that allows for easy and accurate calculation of complex resonances of one-dimensional structures with low losses. As a model problem, we consider a one-dimensional Schroedinger's equation whose potential is a low-energy well surrounded by a thick barrier. The resonance is calculated as a perturbation of the associated bound state when the barrier thickness is infinite. We show that the difference between the resonance and

the bound state frequency is exponentially small in barrier thickness. We devise a simple method to obtain an approximation to the correction. The same approach has been extended to calculate the resonances of a finite periodic structure with a defect. Numerical examples are given to illustrate the main ideas in this work.

17 John Sylvester, *Far Field Support for the Helmholtz Equation*

Thursday, 11:45–12:30.

ABSTRACT: I will discuss the inverse source problem for the Helmholtz equation and the notion of convex scattering support or far field support that Steve Kusiak and I introduced about 10 years ago. The convex far field support is the smallest convex set that supports (carries) a source that radiates that far field. I will review some subsequent developments and extensions of this concept to unions of well separated convex sets, as well as algorithms for computing these sets. All of these developments rely heavily on unique continuation, which makes them general, but does not address well-posedness of the inverse problem. I will outline some work in progress with Roland Griesmaier and Martin Hanke, where, in some regimes, we can replace unique continuation with conditions that depend explicitly on wavelength.

18 Jenn-Nan Wang, *Size estimate problem for the shallow shell system*

Tuesday, 11:00–11:45.

ABSTRACT: this talk, I would like to discuss the problem of estimating the size of an inclusion in the shallow shell system by one pair of boundary measurement. The material property of the inclusion may or may not be given. I will discuss both cases in the talk.

19 Steve Zelditch, *Ergodicity and intersections of geodesics and nodal sets of eigenfunctions*

Thursday, 9:00–9:45.

ABSTRACT: A fundamental problem in the spectral theory of the Laplacian of a Riemannian manifold is to determine the distribution of nodal hypersurfaces (zero sets) of eigenfunctions. Our approach is to study this problem in the complex domain– we analytically eigenfunctions to the complexification of a real analytic Riemannian manifold and study the complex zeros. When the geodesic flow is ergodic we can determine the distribution of the discrete set of intersection points of geodesics and nodal sets.

20 Ting Zhou, *Quantitative thermo-acoustics and related problems.*

Tuesday, 11:45–12:30.

ABSTRACT: Thermo-acoustic tomography is a hybrid multi-waves medical imaging modality that aims to combine the good optical contrast observed in tissues with the good resolution properties of ultrasound. Thermo-acoustic imaging consists of two steps: first to reconstruct an amount of electromagnetic radiation absorbed by tissues by solving inverse problems of acoustic waves; secondly to quantitatively reconstruct the optical property of the tissues from the absorption (reconstructed from the first step), which is an internal functional. We are mostly interested in the second step and show some uniqueness and stability results for the full Maxwell's system models under the assumption that the parameter is small, and the uniqueness, stability and reconstruction results for the scalar model. The key ingredient in showing the second result is the complex geometric optics (CGO) solutions.

A Inverse Boundary Value Problems

Organizers: Rakesh and Mikko Salo

A.1 **Fioralba Cakoni,** *Nonlinear Integral Equations for Inverse Problems in Corrosion Detection from Partial Cauchy Data*

Monday, 2:30–3:00, Emerald Bay C.

ABSTRACT: We consider the inverse problem to recover a part Γ_c of the boundary of a simply connected planar domain D from a pair of Cauchy data of a harmonic function u in D on the remaining part $\partial D \setminus \Gamma_c$ when u satisfies a homogeneous impedance boundary condition on Γ_c. Our approach is based on a system of nonlinear integral equations. As a byproduct, these integral equations can also be used for the problem to extend incomplete Cauchy data and to solve the inverse problem to recover an impedance profile on a known boundary curve. We extend our approach to reconstructing simultaneously both the inaccessible boundary Γ_c and impedance function. We present the mathematical foundation of the method and illustrate its feasibility by numerical examples. This is joint work with Rainer Kress.

A.2 **Boaz Haberman,** *Calderón's problem for low regularity conductivities*

Thursday, 4:00–4:30, Doheny Beach A.

ABSTRACT: Following an idea of Calderón, Sylvester and Uhlmann proved uniqueness in the inverse conductivity problem for smooth conductivities by constructing lots of complex geometrical optics solutions. Much work has gone into reducing the regularity assumptions on the conductivity. By constructing these CGO solutions in specialized function spaces (inspired by Bourgain's $X^{s,b}$ spaces) this result can be extended to C^1 conductivities. This talk is based on joint work with Daniel Tataru.

A.3 **Yaroslav Kurylev,** *Inverse Problem of Electro-magneto-encephalography in the 3-shell Model*

Monday, 3:30–4:00, Emerald Bay C.

ABSTRACT: This work (joint with A. S. Fokas) is devoted to the uniqueness and reconstruction problem of electro-magneto-encephalography. We assume the so-called 3-shell model of the brain consisting of 3 concentric spheres with different values of electric permittivity. In this model we fully describe the non-uniqueness of the recovery of the internal current from the electric and magnetic measurements outside the skull and provide a reconstruction algorithm in the case of minimizing a proper functional.

A.4 **Jennifer Mueller,** *A direct D-bar reconstruction algorithm for recovering a complex conductivity in 2-D*

Monday, 3:00–3:30, Emerald Bay C.

ABSTRACT: A direct reconstruction algorithm for complex conductivities in $W^{2,\infty}(\Omega)$, where Ω is a bounded, simply connected Lipschitz domain in $\mathbb{R}^2$, is presented. The algorithm constitutes the first D-bar method for the reconstruction of conductivities and permittivities in two dimensions. Reconstructions of numerically simulated chest phantoms with discontinuities at the organ boundaries are presented as well as reconstructions from experimental data.

A.5 Alberto Ruiz, *Stability of Calderon problem with partial Data*

Thursday, 2:30–3:00, Doheny Beach A.

ABSTRACT: In a joint work with Dos Santos Ferreira and P. Caro, we prove a stability estimate for Radon transform and its appplication to the stability of the Caldern Problem with partial data.

A.6 Valeri Serov, *Scattering solutions for the magnetic Schrödinger operator. Backscattering Born approximation*

Thursday, 3:30–4:00, Doheny Beach A.

ABSTRACT: We consider in $\mathbb{R}^n (n = 2, 3)$ a magnetic Schrödinger operator

$$H = -(\nabla + i\vec{W}(x))^2 + V(x), \quad x \in R^n,$$

where the coefficients $\vec{W}(x)$ and $V(x)$ are assumed to be real-valued and satisfy the conditions

$$\vec{W}(x) \in L^\infty(R^n), \quad n \geq 2,$$

and

$$\nabla\vec{W}, V \in L^p(R^n), \quad n = 3, \quad 3 \leq p \leq \infty; \quad n = 2, \quad 2 < p \leq \infty.$$

It is assumed also special behavior for V and $\vec{W}$ at the infinity.

We prove the existence of the scattering solutions to the corresponding magnetic Schrödinger equation which belong to the weighted Sobolev space $H^1_{-\delta}(R^n)$ with $\delta > \frac{1}{2}$. As a consequence of this we are able to define the backscattering Born approximation and estimate the first nonlinear term in the Born series.

A.7 Leo Tzou, *The Aharonov-Bohm Effect and the Calderón Problem for Connection Laplacians*

Thursday, 2:00–2:30, Doheny Beach A.

ABSTRACT: The Aharonov-Bohm effect is a quantum mechanical phenomenon where electrons passing through a region of vanishing magnetic field gets scattered due to topological effects. It turns out that this phenomenon is closely related to the cohomology of forms with integer coefficients. We study this relationship from the point of view of the Calderón problem and see that it can be captured in how Cauchy data of the connection laplacian determines uniquely the holonomy representation of the connection.

B Transformation Optics and Cloaking

Organizers: Ting Zhou and Matti Lassas

B.1 **Hoai-Minh Nguyen,** *Approximate cloaking using transformation optics and negative index materials*

Monday, 3:30–4:00, Emerald Bay B.

ABSTRACT: In this talk, I will discuss some analysis on the approximate cloaking using transformation optics and negative index materials.

B.2 **Ulf Leonhardt,** *Perfect imaging with positive refraction*

Friday, 2:00–2:30, Doheny Beach A.

ABSTRACT: Perfect imaging has been believed to rely on negative refraction, but here we show that an ordinary positively-refracting optical medium may form perfect images as well. In particular, we establish a mathematical proof backed up by experimental evidence that Maxwell's fish eye in two-dimensional integrated optics makes a perfect instrument with a resolution not limited by the wavelength of light. We also show how to modify the fish eye such that perfect imaging devices can be made in practice. Our method of perfect focusing may also find applications outside of optics, in acoustics, fluid mechanics or quantum physics, wherever waves obey the two-dimensional Helmholtz equation.

B.3 **Fernando Vasquez,** *Active Exterior Cloaking*

Monday, 2:30–3:00, Emerald Bay B.

ABSTRACT: We present a way of using active sources to hide objects from a known incident field. The active sources cancel out the incident field in a region while having a small far field. Since very little waves reach objects in the cloaked region, the scattered field is greatly diminished, making the object practically invisible. We recall how to construct such a cloak using a single and double layer potential on a surface (Green's formulas) and then show how the same effect can be achieved using a few multipolar sources that do not completely surround the cloaked region. We illustrate this approach for the Laplace (2D), Helmholtz (2D and 3D) and Maxwell equations.

B.4 **Graeme Milton,** *Cloaking for elasticity*

Monday, 3:00–3:30, Emerald Bay B.

ABSTRACT:

B.5 **Hyundae Lee,** *Mathematical analysis of the anomalous localized resonance*

Friday, 3:30–4:00, Doheny Beach A.

ABSTRACT: We give a mathematical analysis of the phenomenon called the anomalous localized resonance. We consider a structure with a layer of metamaterial, which has negative relative permittivity with a small loss parameter. Using layer potential and symmetrization techniques, we show that the solution of the conductivity problem with a source term satisfying an appropriate condition diverges pointwisely in a region exterior to the metamaterial structure as its loss parameter goes to zero. This localized blow-up is directly linked to the cloaking of some kinds of sources.

B.6 Yaroslav Kurylev, *Manifolds of bounded geometry and stability of inverse problems*

Friday, 2:30–3:00, Doheny Beach A.

ABSTRACT: We consider stability of an inverse spectral problem with local data on a class of Riemannian manifolds of bounded sectional curvature and diameter. We prove the uniform stability of this problem with respect to the metric-measure topology. This result deals with discreet data and, in principal, does not depend upon (unknown) dimensionality of the manifolds. When dealing with the local stability, we show it in Holder-type classes. (Joint with M. Lassas and Takao Yamaguchi)

B.7 Mikyoung Lim, *Enhancement of near-cloaking using multilayer structures*

Friday, 4:00–4:30, Doheny Beach A.

ABSTRACT: In this talk, we discuss a method of constructing very effective near-cloaking structures for the conductivity and the scattering problem. These new structures are, before using the transformation optics, layered structures and are designed such that their first generalized polarization tensors or scattering coefficients vanish. Any target inside the cloaking region has near-zero boundary or scattering cross section measurements. We show analytically and numerically that this new multilayer structures significantly enhance the cloaking effect. We also numerically show that this new construction is quite robust with respect to random fluctuations of the material parameters around their theoretical values.

C Inverse Problems in Geometry

Organizers: Vladimir Sharafutdinov and Todd Quinto

C.1 Eric Todd Quinto, *The Microlocal Analysis of some curvilinear Radon transforms*

Tuesday, 2:00–2:30, Doheny Beach B.

ABSTRACT: We will use microlocal analysis to understand two problems in tomography. The models are Radon transforms over complexes of curves, and our reconstruction algorithms are of derivative backprojection type.

We will describe joint work of Felea [SIAM J. Math. Anal. 43(3) 2011, 1145-1157] for slant-hole SPECT and conical-tilt electron tomography in which we explain why added singularities can be mitigated somewhat by choosing a good differential operator in the algorithm in [Local Tomography in 3-D SPECT, joint with Tufts undergraduate researchers Tania Bakhos and Sohhyun Chung, in Mathematical Methods in Biomedical Imaging and Intensity-Modulated Radiation Therapy (IMRT), Yair Censor, Ming Jiang, and Alfred K. Louis, editors, Edizioni della Normale, CRM Series, Pisa, Italy, 2008, pp. 321–348]. This work is an application of fundamental results of Greenleaf and Uhlmann.

Then, we present work with Hans Rullgård [submitted] on a complex of curves in $\mathbb{R}^3$ that is a model for electron microscope tomography with large fields of view. This complex is not admissible in general, so the Greenleaf-Uhlmann theory does not apply, so we use Guillemin's original framework.

C.2 Yernat Assylbekov, *Some integral geometry problems on Finsler and Riemannian surfaces*

Friday, 2:30–3:00, Doheny Beach B.

ABSTRACT: We consider the scalar and vector integral geometry problems on surfaces. Given a

compact oriented Finsler surface with smooth boundary, we prove injectivity of the scalar and vector X-ray transform over a general family of curves running between boundary points, under a natural condition that results in the no conjugate points condition in the case when the curves in question are geodesic lines. The main result generalizes Mukhometov's theorem in several directions. This work was done jointly with Nurlan S. Dairbekov.

Also, we consider the tensor geodesic X-ray transform with attenuation given as the combination of a smooth complex function and a smooth 1-form on a simple Riemannian surface. We show that the attenuated tensor X-ray transform is injective modulo the natural obstruction.

C.3 **Francois Monard,** *The inverse conductivity problem with power densities in general dimension.*

Tuesday, 2:30–3:00, Doheny Beach B.

ABSTRACT: In this talk, I will review recent results on the inverse conductivity problem with internal data functionals of "power density" type, a problem that is motivated by hybrid medical imaging methods. The main results are that, provided that one can generate solutions of the conductivity equation that satisfy certain qualitative properties (linear independence of their gradients, among others), all of the anisotropic tensor is explicitely and rather stably reconstructible. In particular, if we decompose the conductivity into the product of a scalar part with a rescaled anisotropic structure, we establish that (i) the scalar part can be inverted explicitely with no loss of derivative w.r.t. the data, and (ii) the anisotropic structure can also be reconstructed explicitely, with the loss of one derivative. The reconstruction algorithms rely heavily on linear algebraic concepts as well as calculus using a connection, as one aims at reconstructing a certain frame as an intermediate step toward reconstruction in certain cases.

This is joint work with Guillaume Bal.

C.4 **Leonid Pestov,** *On determining a conformal euclidean metric by its copy*

Friday, 2:00–2:30, Doheny Beach B.

ABSTRACT: Let $M \subset \mathbb{R}^n, n \geq 2$ be a compact set with smooth boundary ∂M and with conformal-euclidean metric $(g_e)_{ij} = \rho\delta_{ij}$. Let $U \subset M$ be an open set such that $\Gamma = \partial U \cap \partial M$ is nonempty. Let $f : \mathbb{R}^n \supset V \to U$ be diffeomorphism, which is identical for $y \subset f^{-1}(\Gamma)$ (for simplicity). Consider problem: given metric $g = f^* g_e$ and $df|_\Gamma$ to find f (g_e is unknown). This problem is close to Yamabe problem for the case of zero curvature and may be reduced to the Cauchy problem for elliptic equation $a_n \Delta_g u + Ru = 0$, where R is the scalar curvature of metric $f^* g$, a_n is some constant [LP]. However, we give another solution, based on conformal Killing's vector fields. The main advantage of our approach is that it gives a stable solution for dimension $n > 2$. This problem appears in inverse dynamical problems for isotropic mediums, when a copy of a conformal euclidean metric in semigeodesic coordinates is determined and it needs to return back to the original isothermal coordinates [B].

Vector field u is called conformal Killing's vector field if it satisfies Killing's equation $Ku = \sigma\nabla u - g\delta u/n = 0$, where $\sigma\nabla$ is the symmetric part of covariant derivative, δ is the divergence. All conformal euclidean metrics admit the same set of conformal Killing's vector fields u. For two-dimensional case $u = (u^1, u^2)$, where u^1 and u^2 are conjugate harmonic functions. In the case of $n > 2$ contravariant components of u are given by $u^i(x) = a_0 x^i + (Ax)^i - b^i|x|^2 + 2x^i(b, x) + c^i$,where a_0 is a constant, A is a $n \times n$ skew-symmetric matrix , b and c are vectors in $\mathbb{R}^n$.

The procedure of determining f is the following. Let $e_{(j)}, j = 1, ..., n$ are standard basis vectors in $\mathbb{R}^n$. They are conformal Killing's vector fields for any conformal euclidean metric g_e. Then $u_{(j)} = f^* e_{(j)},\ j = 1, ..., n$ are conformal Killing's vector fields for metric $g = f^* g_e$. Given metric g and trace $df|_\Gamma$ one can find fields $u_{(j)}$ from Killing's equation. Then equalities $u_{(j)} = f^* e_{(j)}, j = 1, ..., n$ determines df and (since $f|_\Gamma = id$) we get f. For $n = 2$ the problem $Ku = 0,\ u|_\Gamma = u_0$ is ill-posed. For $n > 2$ there is a estimate

$$||u||^2_{L^2(U)} \leq C(\rho)||u_0||_{C^2(\Gamma)},$$

where constant $C(\rho)$ depends on positive constants c_1, c_2, $c_1 < \rho(x) < c_2$.

References

[LP] J.M. Lee, T.H. Parker, The Yamabe problem, *Bull. Amer. Math. Soc.*, **17**, 1987, 37-91.

[B] M.I. Belishev, Recent progress in the boundary control method, *Inverse Problems*, **23**, No. 5, 2007, 1-67.

C.5 Aleksander Denisiuk, *On support theorems for the X-Ray transform with incomplete data*

Tuesday, 4:00–4:30, Doheny Beach B.

Abstract: A. Greenleaf and G. Uhlmann in their article *Non-local inversion formulas in integral geometry,* noticed that canonical relation, associated with the X-ray transform on n-dimensional line complex is almost everywhere canonical graph. This observation was exploited by various authors. Specifically, J. Boman and E. T. Quinto in *Support theorems for radon transforms on real analytic line complexes in three space,* used similar fact to prove uniqueness theorems for the X-ray transform on *admissible* complexes of lines. We will discuss the notion of admissibility, its relation with microlocal analysis and support theorems for the X-ray transform. Specifically, we will take a look at the Boman-Quinto support theorems for the X-ray transform on real analytic line complexes in three-space and investigate ways to generalize this theorems to higher dimensions.

C.6 Alexandre Jollivet, *Inverse scattering in classical mechanics*

Friday, 3:30–4:00, Doheny Beach B.

Abstract: We consider the inverse scattering problem for the multidimensional relativistic Newton equation in an external short-range and smooth electromagnetic (or gravitational) field. We briefly recall uniqueness results for the inverse scattering at high energies, and uniqueness and stability results for the inverse scattering problem at fixed energy when the external field is compactly supported. Then we present a new uniqueness result for the latter problem when the external (electric or gravitational) field is spherical symmetric outside a bounded domain.

C.7 Clayton Shonkwiler, *The Dirichlet-to-Neumann map for differential forms*

Tuesday, 3:30–4:00, Doheny Beach B.

Abstract: The classical Dirichlet-to-Neumann map is an operator on functions on the boundary of a Riemannian manifold which arises in the problem of Electrical Impedance Tomography. I will discuss a generalization of this operator to differential forms and how it can be used to recover geometric and topological information about the manifold.

C.8 Katya Krupchyk, *Inverse boundary problems for magnetic Schrödinger operators with continuous magnetic potentials.*

Friday, 4:00–4:30, Doheny Beach B.

ABSTRACT: We show that the knowledge of the set of the Cauchy data on the boundary of a C^1 bounded open set in $\mathbb{R}^n$, $n \geq 3$, for the Schrödinger operator with continuous magnetic and bounded electric potentials determines the magnetic field and electric potential inside the set uniquely. Our approach is based on a Carleman estimate for the magnetic Schrödinger operator. This is joint work with Gunther Uhlmann.

D Hybrid Methods (Multi-Physics, Multi-Wave) in Medical Imaging

Organizers: Guillaume Bal and Shari Moskow

D.1 Sebastian Imperiale, *Reconstruction of coefficients in elliptic equations from knowledge of their solutions : Scalar and elastic cases.*

Wednesday, 4:30–5:00, Emerald Bay B.

ABSTRACT: In this talk we discuss the problem of the quantitative identification of the coefficients of some elliptic equations from the knowledge of their solutions inside a domain of interest. In particular, we will focus on the novel reconstruction algorithms that have been recently developed in: [1] G. Bal, G. Uhlmann, (2012) Reconstruction of coefficients in scalar second-order elliptic equations from knowledge of their solutions. Submitted [2] G. Bal, G. Uhlmann, (2012) Reconstructions for some coupled-physics inverse problems. Applied Mathematics Letters

Considering the second-order elliptic equation of conductivity, we will discuss the theoretical framework introduced in these articles together with the proposed reconstruction formulae in the cases where the assumed anisotropic conductivity is known or unknown. A regularization approach to deal with noisy measurements will be proposed and discussed. Numerical results involving synthetic data will be presented. Finally, an extension of this approach to isotropic elasticity systems will be presented and further illustrated with a set of numerical results. (Joint work with Guillaume Bal, Cedric Bellis and Francois Monard.

D.2 Chenxi Guo, *Linearized internal functionals for anisotropic conductivities*

Wednesday, 5:00–5:30, Emerald Bay B.

ABSTRACT: We consider inverse anisotropic conductivity problem with linearized power density measurements in $\mathbb{R}^n$. In the first part, we will show that, with only principle symbols of pseudo-differential operator, one can not control all directions. Then we pass to the symbols of order -1 and show that we can reconstruct γ modulo a compact operator. In the second part, we will construct γ microlocally and show that $n+1$ solutions and $\frac{n(n+3)}{2}$ measurements will be enough.

D.3 Leonid Kunyansky, *A mathematical model and inversion procedure for Magneto-Acousto-Electric Tomography*

Tuesday, 2:00–2:30, Doheny Beach A.

ABSTRACT: Magneto-Acousto-Electric Tomography is a novel hybrid modality that represents

a stable, high-resolution alternative to the Electrical Impedance Tomography. We analyze existing mathematical models of MAET, and present a general procedure for solving the associated inverse problem. For a general acquisition service the procedure consists of a synthetic focusing step, followed by a solution of the Neumann problem for the Laplace equation, and solution of the Poisson equation. If the acquisition surface is a cube, one obtains an explicit Fourier series solution resulting in a fast reconstruction algorithm. Our analysis and numerical simulations show that MAET is a stable technique capable of producing high-resolution images even in the presence of significant noise in the data.

D.4 Shari Moskow, *Local inversions in ultrasound modulated optical tomography*

Tuesday, 2:30–3:00, Doheny Beach A.

ABSTRACT: We consider the problem of the simultaneous recovery of both the absorption and diffusion coefficients with internal data in a form that is of power density type. We assume we have some known background coefficients, and examine the corresponding linearized problem. This linearized problem can be expressed as a fourth order system, and we examine its stability and injectivity properties. (Joint work with Guillaume Bal.)

D.5 Adrian Nachman, *convergent algorithm for the hybrid problem of reconstructing conductivity from minimal interior data*

Wednesday, 3:00–3:30, Emerald Bay B.

ABSTRACT: We consider the hybrid problem of reconstructing the isotropic electric conductivity of a body from interior Current Density Imaging data obtainable using MRI measurements. We only require knowledge of the magnitude $|J|$ of one current for a given voltage f on the boundary.

The corresponding voltage potential is a minimizer of a weighted least gradient problem. We focus to a large extent on the dual variational problem. This leads naturally to an alternating split Bregman algorithm, for which we prove convergence. The dual problem also turns out to yield a novel method to recover the full vector field J from knowledge of its magnitude, and of the voltage f on the boundary. We then present several numerical experiments that illustrate the convergence behavior of the proposed algorithm. This is joint work with Amir Moradifam and Alexandre Timonov.

D.6 Kui Ren, *Uncertainties in quantitative photoacoustic tomography*

Tuesday, 3:30–4:00, Doheny Beach A.

ABSTRACT: In quantitative photoacoustic tomography (qPAT), we are interested in reconstructing the absorption, the scattering and the Grüneisen coefficients from measured acoustic signals measured on the boundary of the domain. Other medium parameters, such as the acoustic wave speed and the acoustic attenuation are assumed to be known exactly. In practice, however, these parameters are not known exactly, but only up to certain accuracy. This introduces uncertainties in the inverse problem. We perform a systematic numerical study on the effect of these uncertainties on the quantitative reconstruction of the coefficients of interests.

D.7 Alex Tamasan, *Conductivity imaging via some minimum weighted total variation problems*

Wednesday, 3:30–4:00, Emerald Bay B.

ABSTRACT: This talk concerns the hybrid problem of conductivity reconstruction from one

interior measurement of the magnitude of the current density field and boundary values of voltage and/or currents. The voltage potential generated from the boundary is of minimum total variation with respect to the weight given by the interior data. I will address various minimization methods and address their convergence.

D.8 **Faouzi Triki,** *Vibration potential tomography*

Tuesday, 4:00–4:30, Doheny Beach A.

ABSTRACT: In vibration potential tomography, ultrasonic waves are focused on regions of small diameter inside a body placed on a static magnetic field. An asymptotic analysis of the DtN of the conduction equation with respect to the size of the perturbation provide the current. The new inverse problem is then to recover the conductivity from the knowledge of the current. In this talk I will discuss the uniqueness and stability of this inverse problem in the case where the conductivity is anisotropic.

E Microlocal Methods

Organizers: Allan Greenleaf and Raluca Felea

E.1 **Andras Vasy,** *Diffraction from conormal singularities*

Wednesday, 3:00–3:30, Emerald Bay C.

ABSTRACT: Waves reflecting/refracting/transmitting from singularities of a metric (e.g. sound speed) satisfy the law of reflection. One expects that if the singularities are sufficiently weak, in terms of differentiability (conormal order) then the reflected singularity is weaker than the transmitted one, in the sense that it is more regular. In this joint work with Maarten de Hoop and Gunther Uhlmann we prove such a result with slightly more regular than C^1 metrics.

E.2 **Melissa Tacy,** *L^p eigenfunction estimates and directional oscillation*

Wednesday, 4:30–5:00, Emerald Bay C.

ABSTRACT: In this talk I will discuss the problem of determining estimates for the L^p growth of eigenfunctions (and quasimodes) of the Laplace-Beltrami operator in situations where the eigenvalue multiplicity is low. To understand this growth we decompose a general quasimode as a sum of directionally localized quasimodes on which we have good L^p control. The problem of general L^p growth then reduces to the study of the interaction between directionally localized pieces.

E.3 **Kiril Datchev,** *Fractal Weyl laws for asymptotically hyperbolic manifolds*

Wednesday, 3:30–4:00, Emerald Bay C.

ABSTRACT: For asymptotically hyperbolic manifolds with hyperbolic trapped sets we prove a fractal upper bound on the number of resonances near the essential spectrum, with power determined by the dimension of the trapped set. This covers the case of general convex cocompact quotients (including the case of connected trapped sets) where our result implies a bound on the number of zeros of the Selberg zeta function in disks of arbitrary size along the imaginary axis. Although no sharp fractal lower bounds are known, the case of quasifuchsian groups, included here, is most likely to provide them. This project is joint work with Semyon Dyatlov.

E.4 Hamid Hezari, *Wave invariants and inverse spectral problems*

Thursday, 2:30–3:00, Doheny Beach B.

ABSTRACT: In this talk we are concerned with inverse spectral problems for Schrodinger operators. We first give new explicit formulas for the wave trace invariants of a semiclassical Schrodinger operator in $\mathbb{R}^n$. We then discuss to what extend the symmetry assumptions on the potential (which were assumed in all the previous results) can be removed. Our goal is to show that no symmetry is needed if we restrict ourselves to a generic class of real analytic potentials.

E.5 Dean Baskin, *Asymptotics of radiation fields in asymptotically Minkowski space*

Thursday, 2:00–2:30, Doheny Beach B.

ABSTRACT: Radiation fields are (appropriately rescaled) limits of solutions of wave equations along light rays. In this talk I will describe a class of (non-static) asymptotically Minkowski space times for which the radiation field is defined and indicate how methods of Vasy can be used to express the asymptotics in terms of the resonances of a related Riemannian problem on an asymptotically hyperbolic manifold. In particular, even on Minkowski space, these methods give a new understanding of the Klainerman-Sobolev estimates. This is joint work with Andras Vasy and Jared Wunsch.

E.6 Matti Lassas/Lauri Oksanen, *An inverse problem for the wave equation with disjoint sources and receivers*

Wednesday, 5:00–5:30, Emerald Bay C.

ABSTRACT: We consider the wave equation on a smooth compact Riemannian manifold (M, g) with boundary and show that acoustic measurements with sources and receivers on disjoints sets $\Sigma, \Gamma \subset \partial M$ determine the manifold uniquely assuming that the wave equation is exactly controllable from the set of sources Σ. Moreover, we show that the exact controllability can be replaced by the strictly weaker spectral condition

$$\lambda_j \leq C \left\| \partial_\nu \phi_j \right\|^2_{L^2(\Sigma)}, \quad j = 1, 2, \ldots,$$

where λ_j are the Dirichlet eigenvalues of (M, g) and $(\phi_j)_{j=1}^\infty$ is an orthonormal basis of eigenfunctions. The talk is based on a joint work with Matti Lassas.

E.7 Cliff Nolan, *Microlocal analysis of radar imaging of a dynamic reflectivity function*

Thursday, 3:30–4:00, Doheny Beach B.

ABSTRACT: We extend the model for scattering of radio waves from a static object to one which is moving. Our approach is related to the recent papers of Cheney and Borden but we consider a more general reflectivity function and a variety of acquisition geometries. We express the scattered waves as the output of Fourier integral operator F that acts on a dynamic reflectivity function. The normal operator F^*F which arises in imaging of the latter function is analyzed and its distribution kernel is found to belong to the class of paired Lagrangian distributions. We are able to apply the results of Marhuenda to show that artifacts arise and are associated to a flow-out from the diagonal in the phase space of space-time. Furthermore, we show how such artifacts can be ameliorated by preprocessing the scattered waves with pseudo-differential operator. This is joint work with Raluca Felea and Romina Gaburro.

E.8 Carlos Montalto, *Stable determination of a simple metric, a covector field and a potencial from the hyperbolic Dirichlet-to-Neumann map*

Thursday, 4:00–4:30, Doheny Beach B.

ABSTRACT: Let (M, g) be a compact Riemmanian manifold with non-empty boundary. Consider the second order hyperbolic initial-boundary value problem

$$\begin{cases} (\partial_t^2 + P(x,D))u &= 0 & \text{in} & (0,T)\times M, \\ u(0,x) = \partial_t u(0,x) &= 0 & \text{for} & x \in M, \\ u(t,x) &= f(t,x) & \text{on} & (0,T)\times \partial M, \end{cases}$$

where

$$P(x,D) = -\frac{1}{\sqrt{\det g}}\left(-\frac{\partial}{\partial x^j} + \mathrm{i}\, b_j\right) g^{ij}\sqrt{\det g}\left(-\frac{\partial}{\partial x^i} + \mathrm{i}\, b_i\right) + q$$

is a first-order perturbation of the Laplace operator $-\triangle_g$ on (M, g). Here b and q are covector field and potential, respectively, in M.

We prove Hölder type stability estimates near generic simple Riemannian metrics for the inverse problem of recovering g, b, and q from the Dirichlet-to-Neumann(DN) map associated, $f \to \partial_\nu u^f - \mathrm{i}\,\langle \nu, b\rangle_g u|_{\partial M\times[0,T]}$ modulo a class of transformations that fixed the DN map.