Problems that led me to Gunther Uhlmann

David Isaacson

RPI

1. Inverse problem in electrocardiography.

2. Inverse boundary value problem for conductivity.

GU

Can we improve the diagnosis and treatment of heart disease?

How does the heart work?

The heart is an electro-mechanical pump.

How does the pump work?

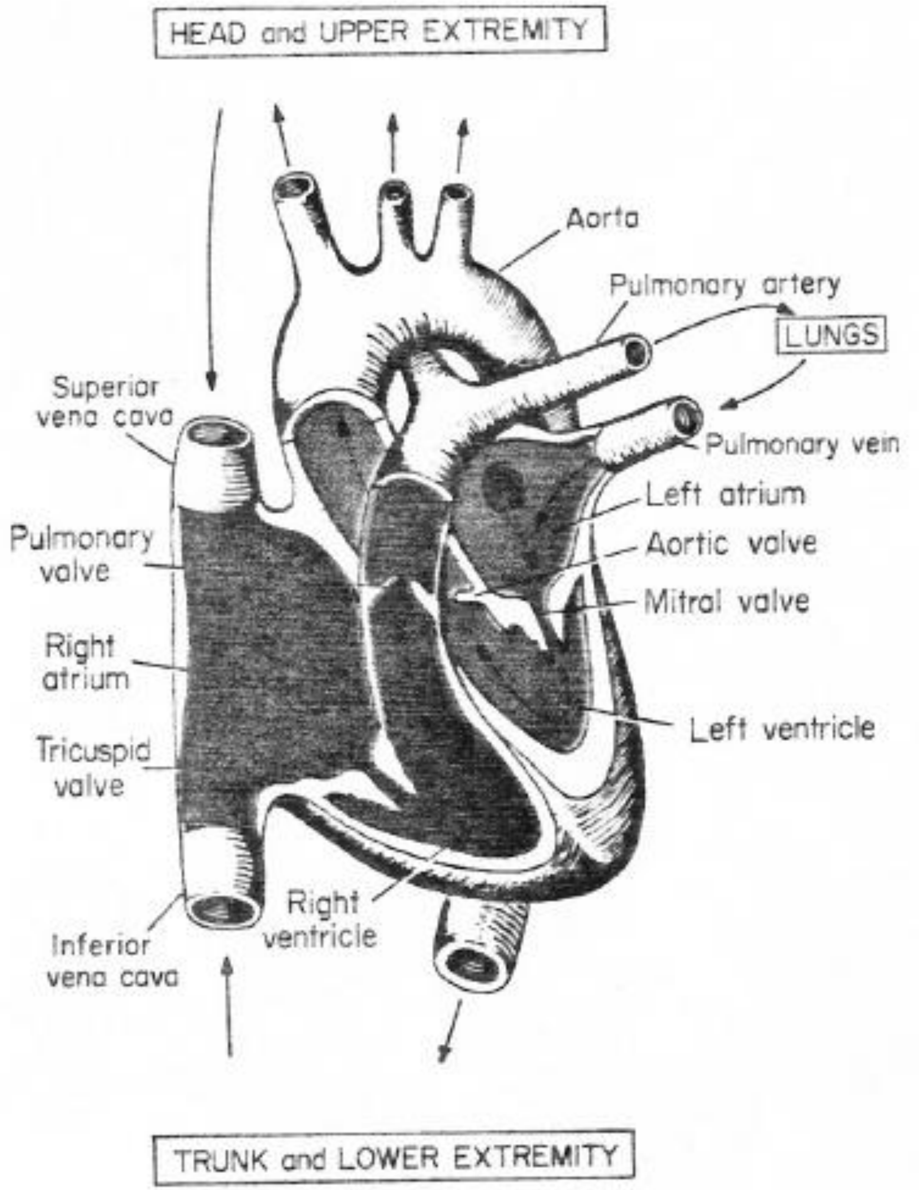
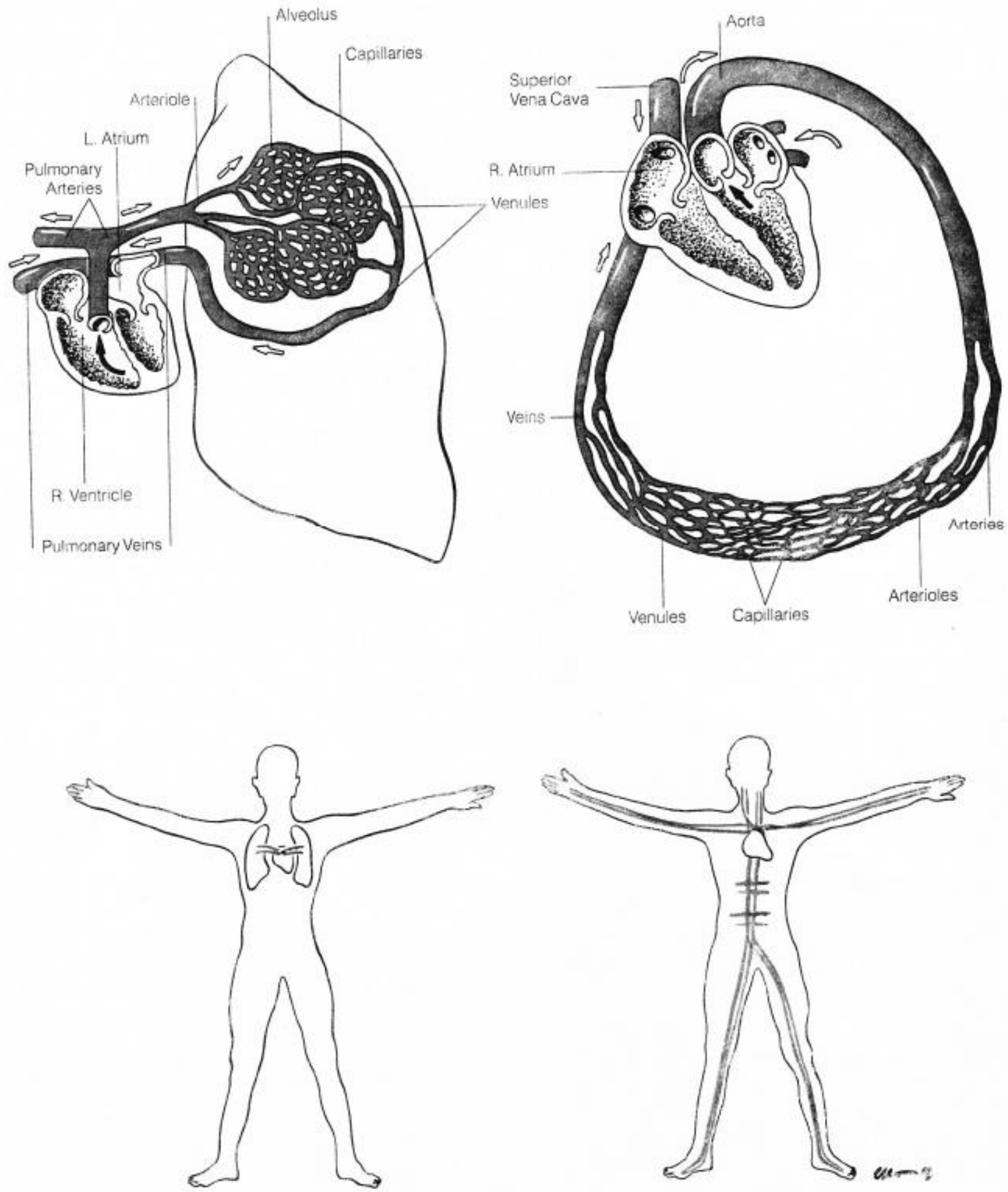


Figure 13–4. Structure of the heart, and course of blood flow through the heart chambers.

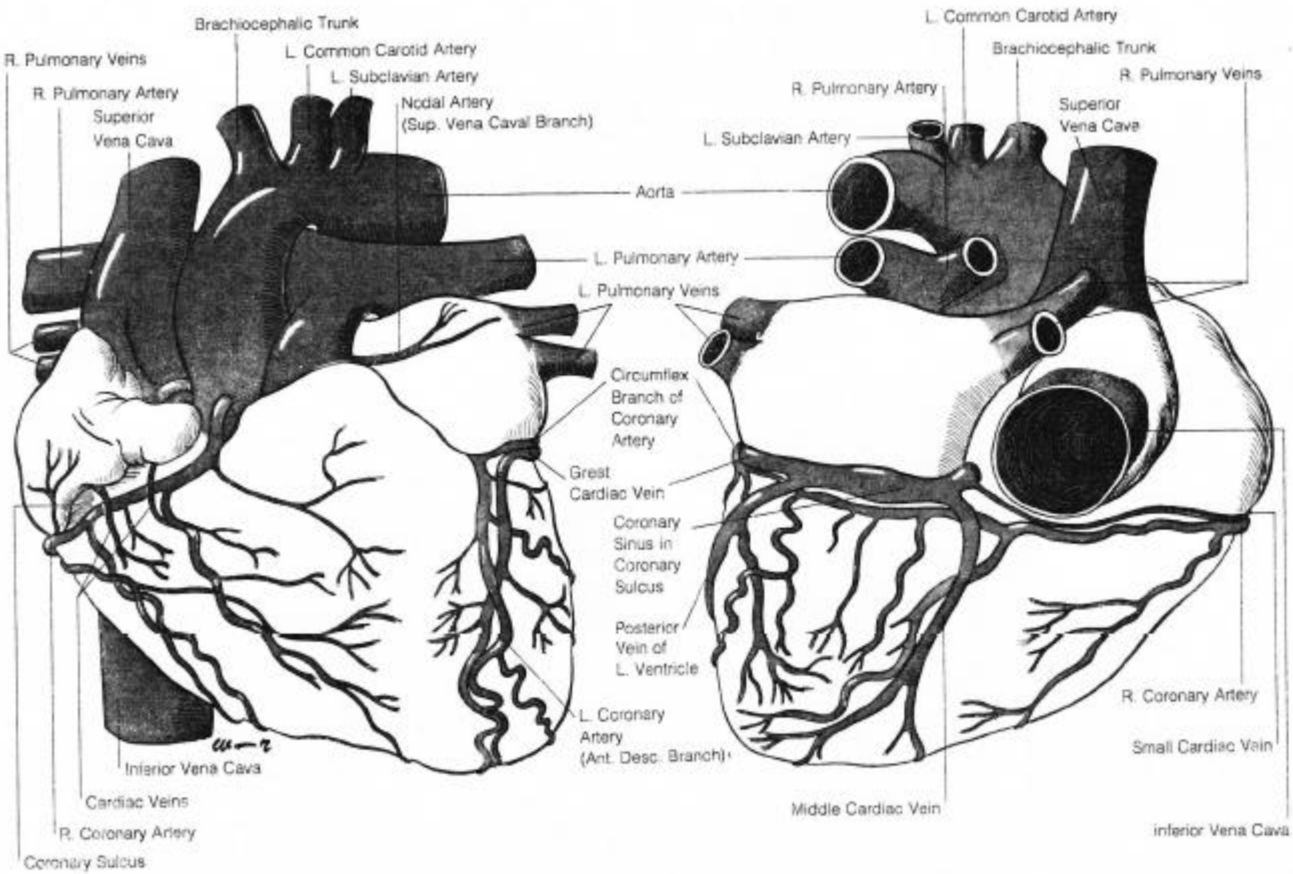


Pulmonary Circulation

Systemic Circulation

FIGURE 2-5
Systemic and Pulmonary Circulation

FIGURE 2-4
Coronary Circulation



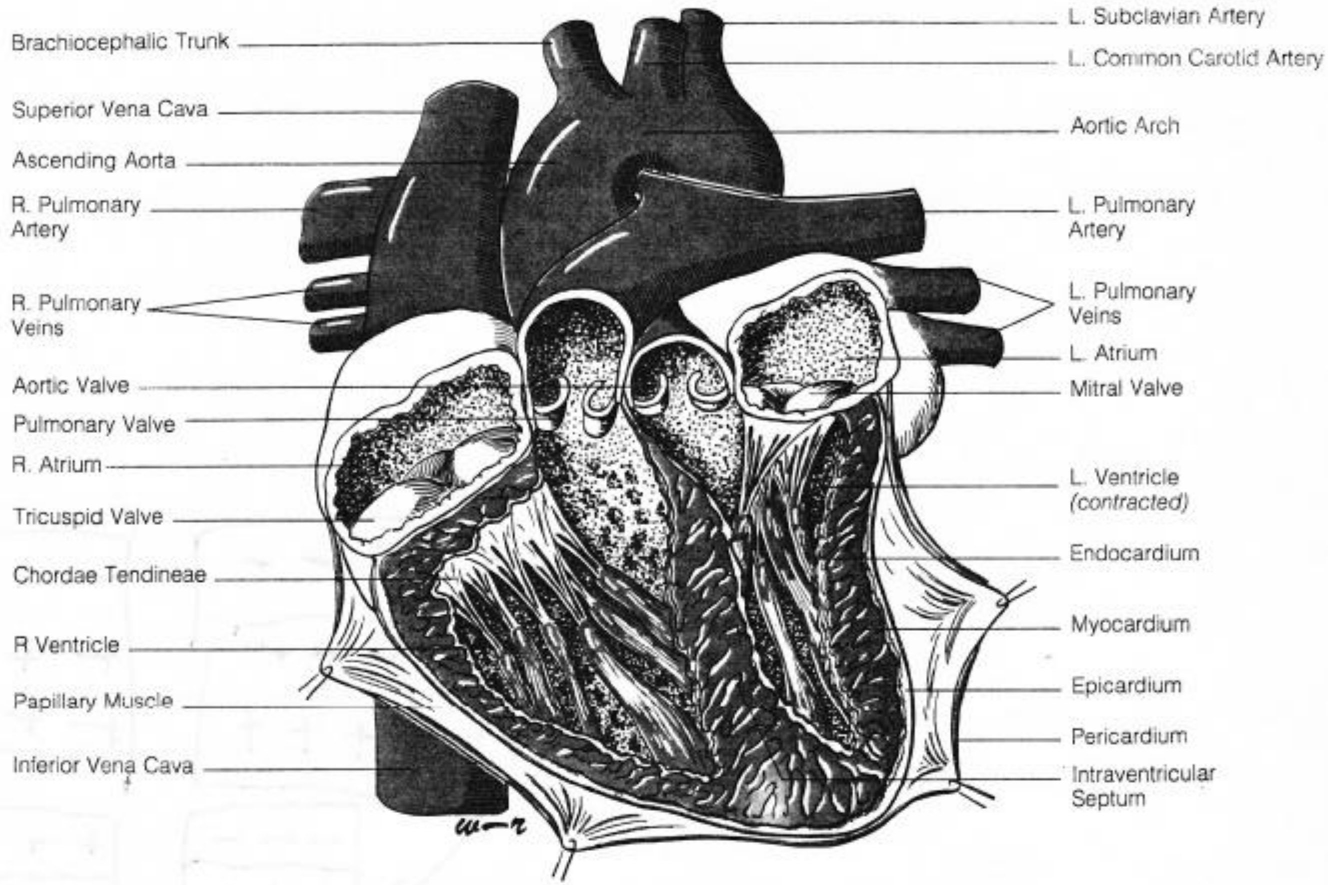


FIGURE 2-3
Cardiac Anatomy

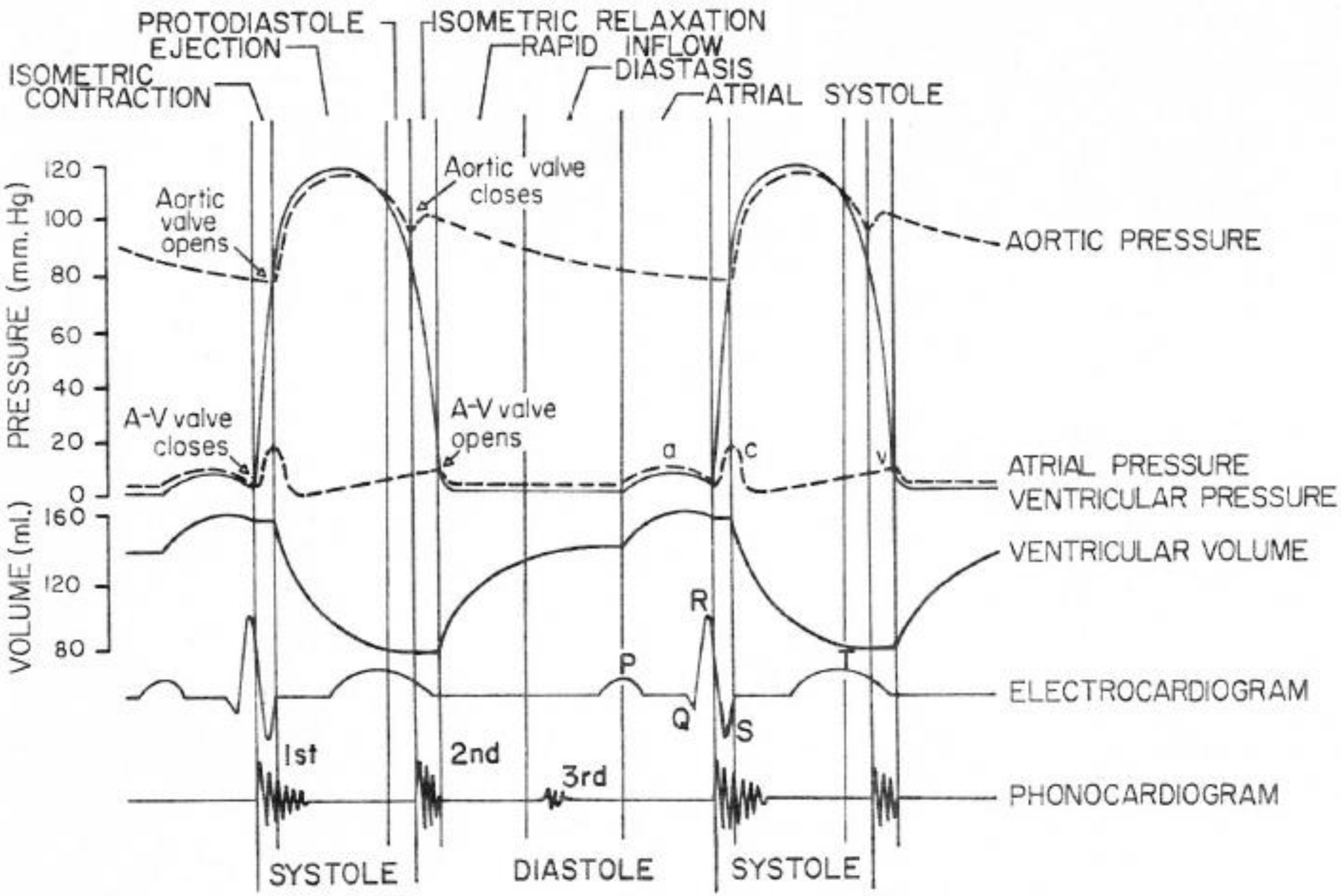
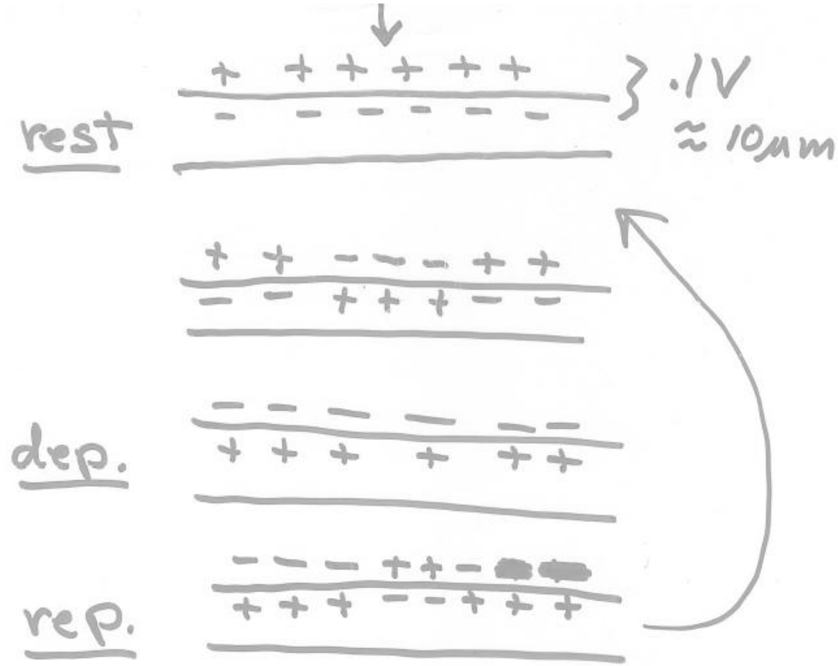


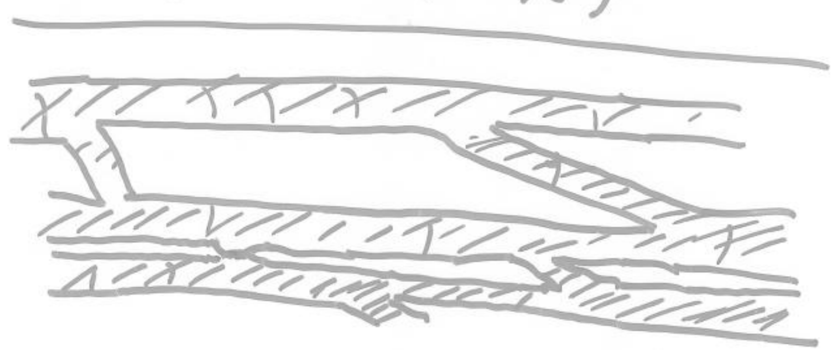
Figure 13–5. The events of the cardiac cycle, showing changes in left atrial pressure, left ventricular pressure, aortic pressure, ventricular volume, the electrocardiogram, and the phonocardiogram.

How does the electrical part work?



Conduction velocities ≈ .5 m/s

(.02 — 4 m/s)



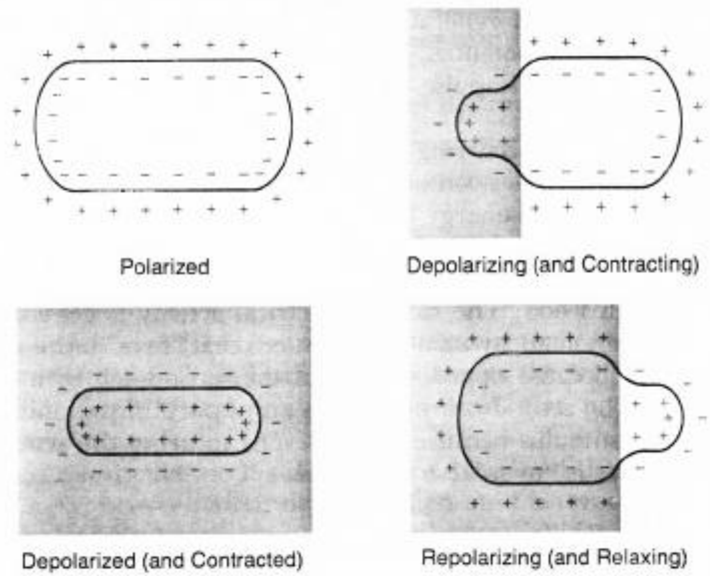
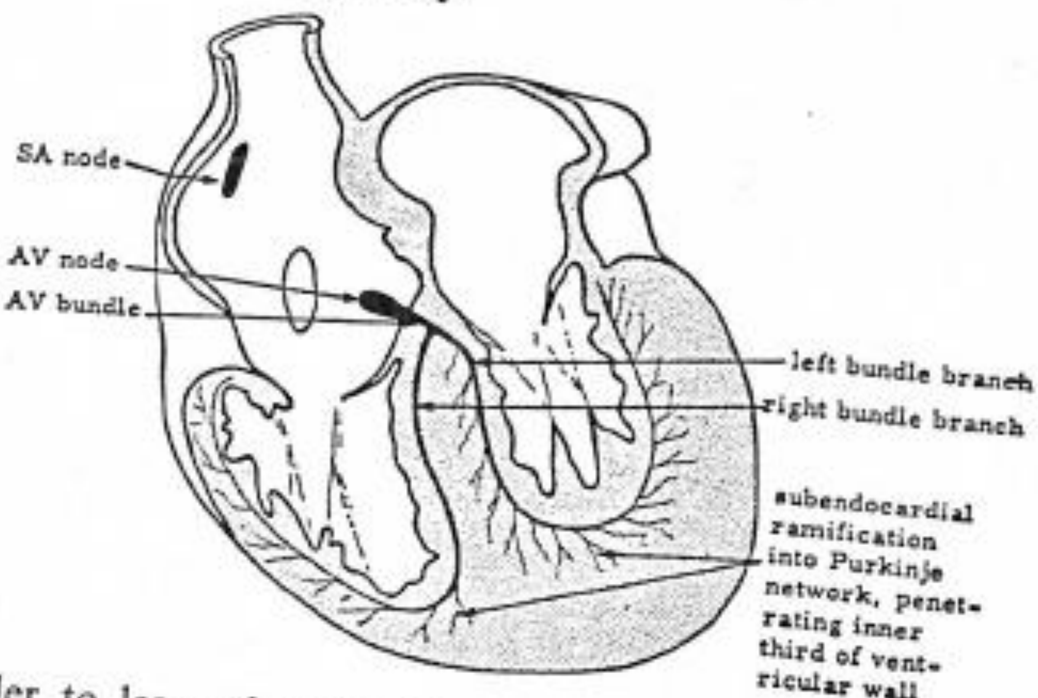


FIGURE 2-7
Cellular Depolarization-Repolarization

carefully.



der to learn the

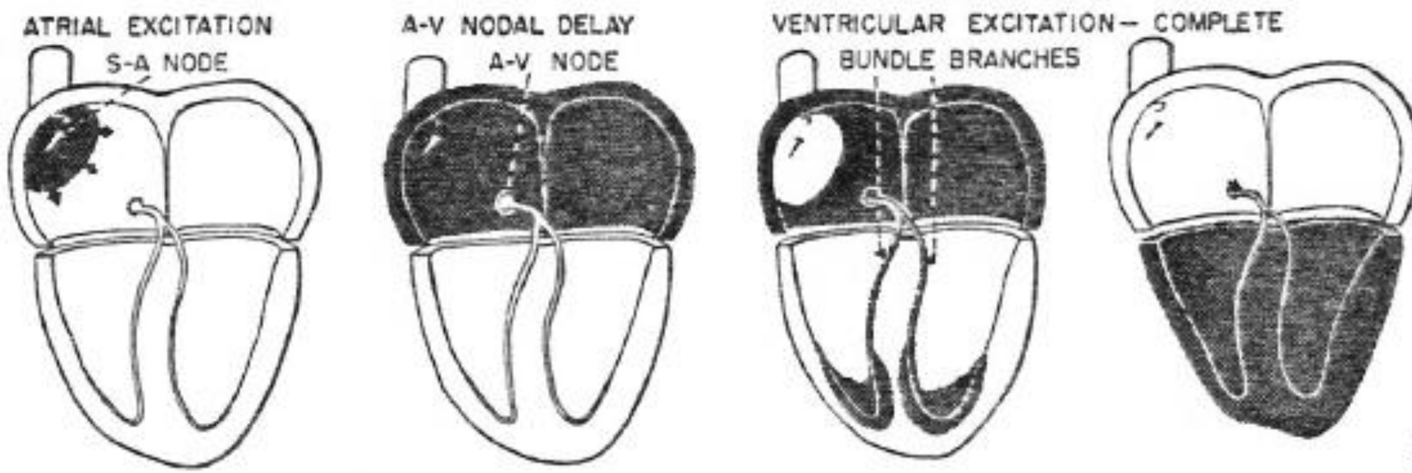


Fig 4

FIGURE 2–12 SEQUENCE OF CARDIAC EXCITATION

Excitation of the heart is normally initiated by an impulse which is generated by the S-A node and which spreads rapidly in all directions through the atrial musculature. After a slight delay at the A-V node, impulses are conducted by the Purkinje system into the ventricles where a wave of excitation spreads from the endocardial surfaces through the ventricular musculature.

How is the electrical function diagnosed?

Electrocardiograms
(ECG, or EKG)
1887 - Waller,
1892 - Einthoven

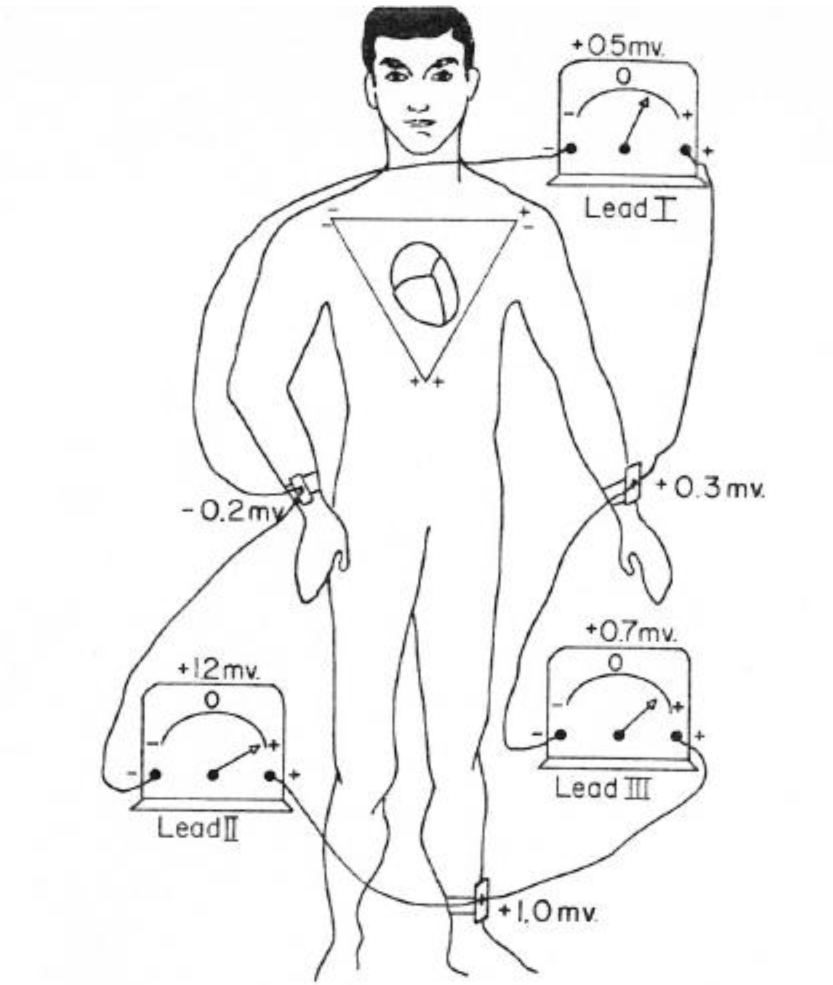
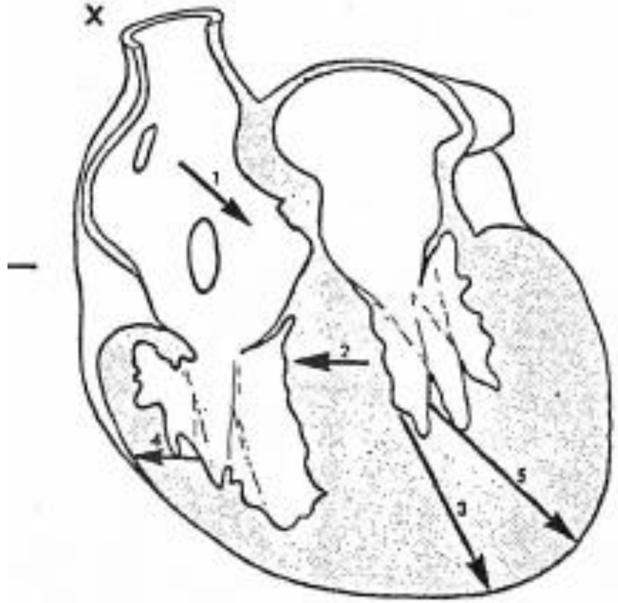
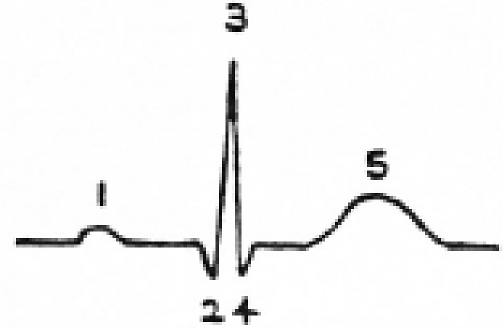
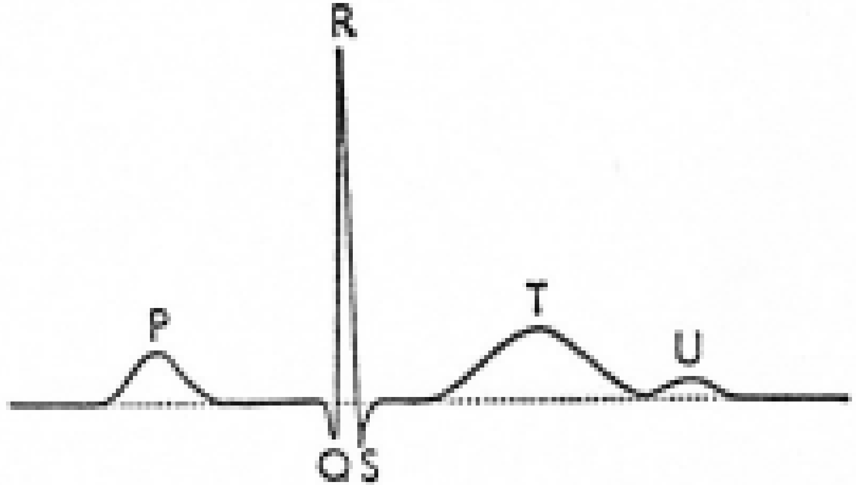
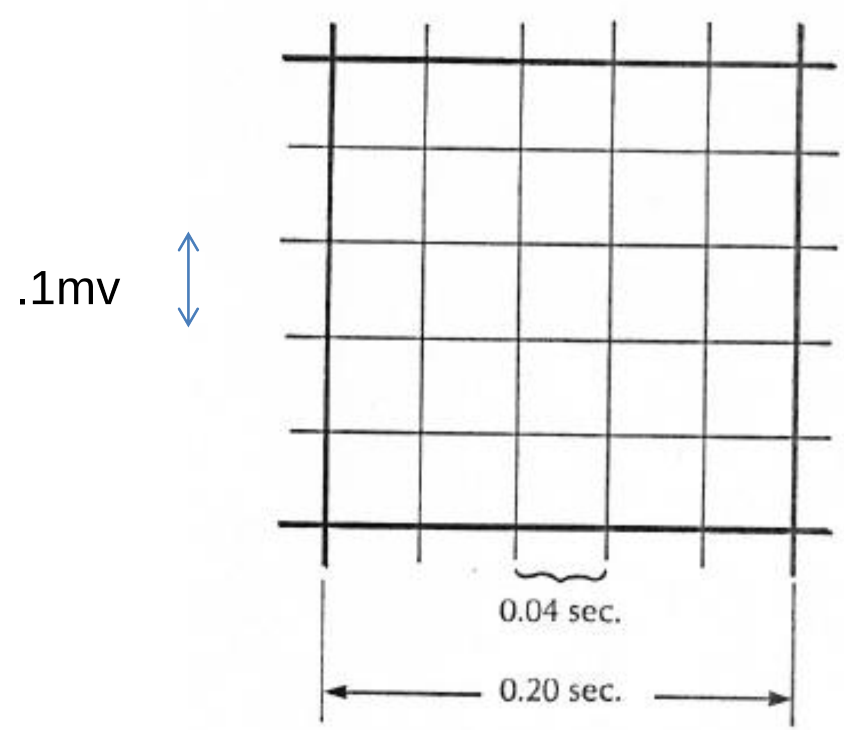


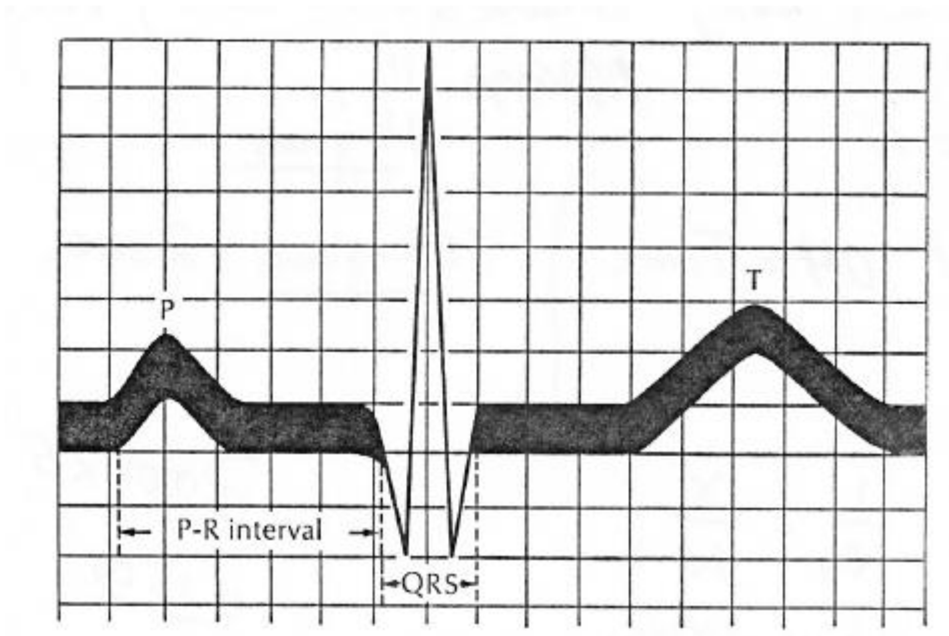
Figure 15–6. Conventional arrangement of electrodes for recording the standard electrocardiographic leads. Einthoven's triangle is superimposed on the chest.

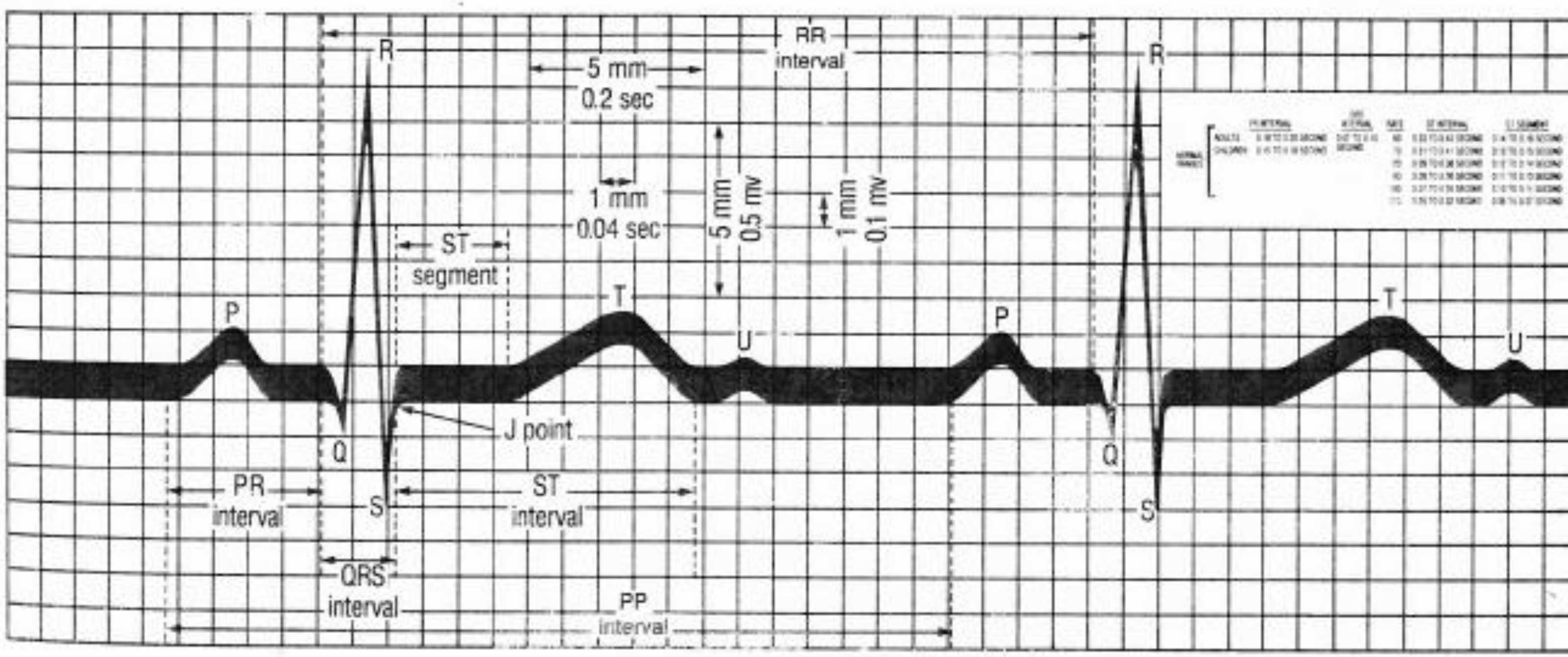



3
1
5
2
4

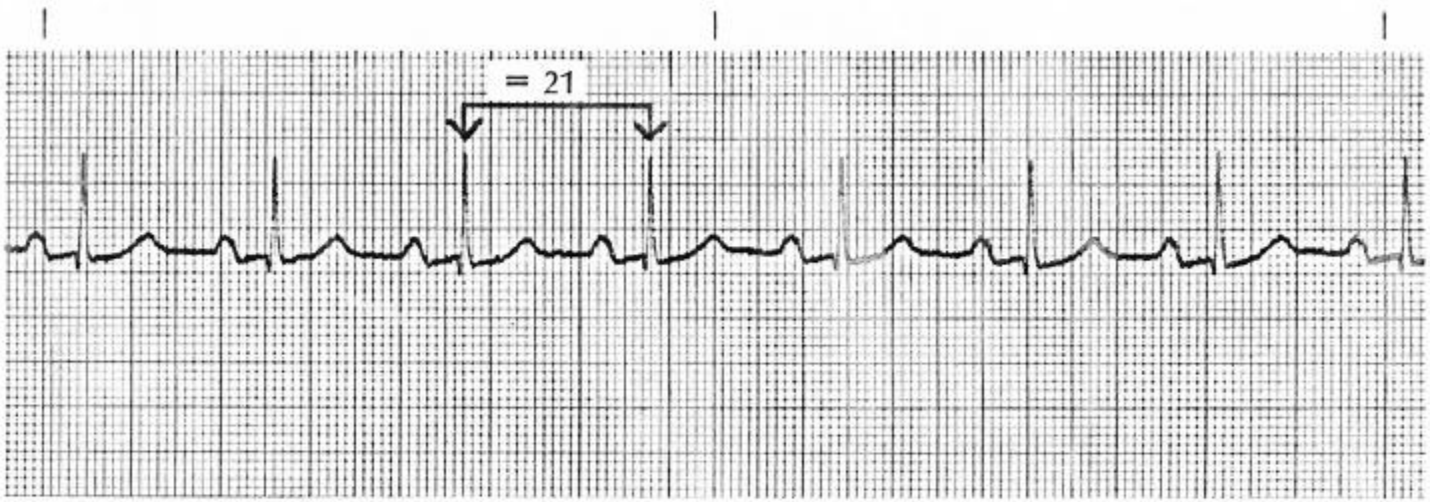

R
P
T
U
Q
S

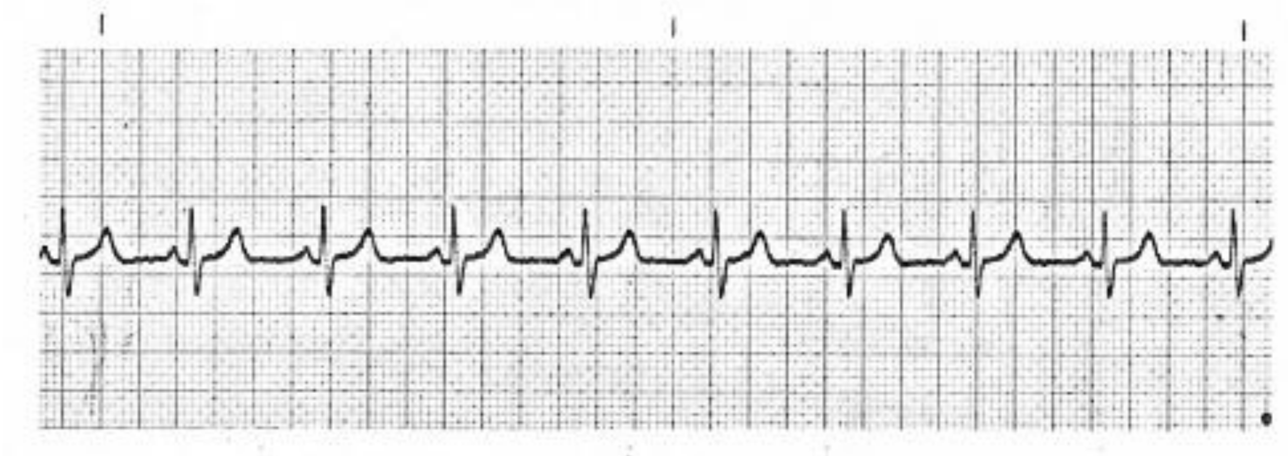

.1mv
0.04 sec.
0.20 sec.


P
T
P-R interval
QRS



RR interval
R
5 mm
0.2 sec
1 mm
0.04 sec
5 mm
0.5 mv
1 mm
0.1 mv
ST segment
P
T
U
Q
S
J point
PR interval
ST interval
QRS interval
PP interval


= 21



Rate 88 Rhythm Regular

P waves OK P-R interval 0.14 sec.

QRS interval 0.08 sec.

Interpretation NSR

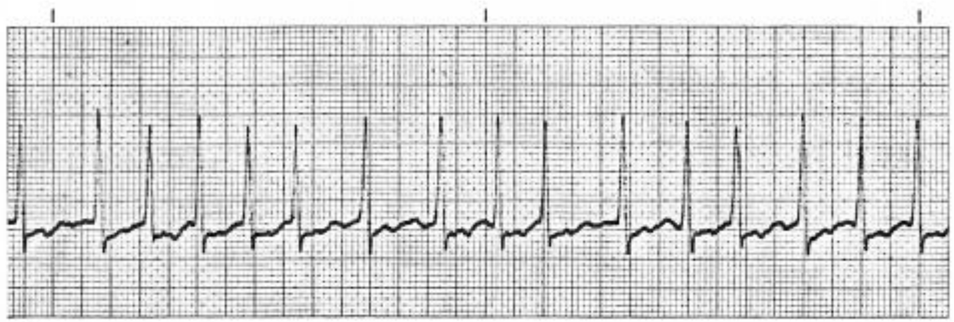


Fig. 8-4

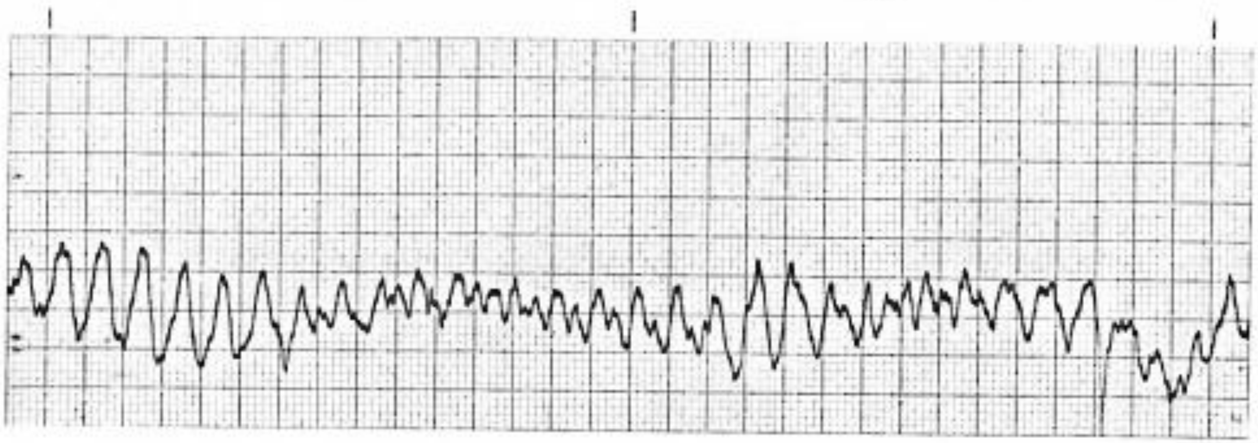
Rate ______________ Rhythm ______________ About 150; Irregularly irregular

P waves ______________ P-R interval ______________ Not apparent; Not measurable

QRS interval ______________ 0.08 sec.

Interpretation ______________ Atrial fibrillation

____



Rate None Rhythm None

P waves None P-R interval Nonexistent

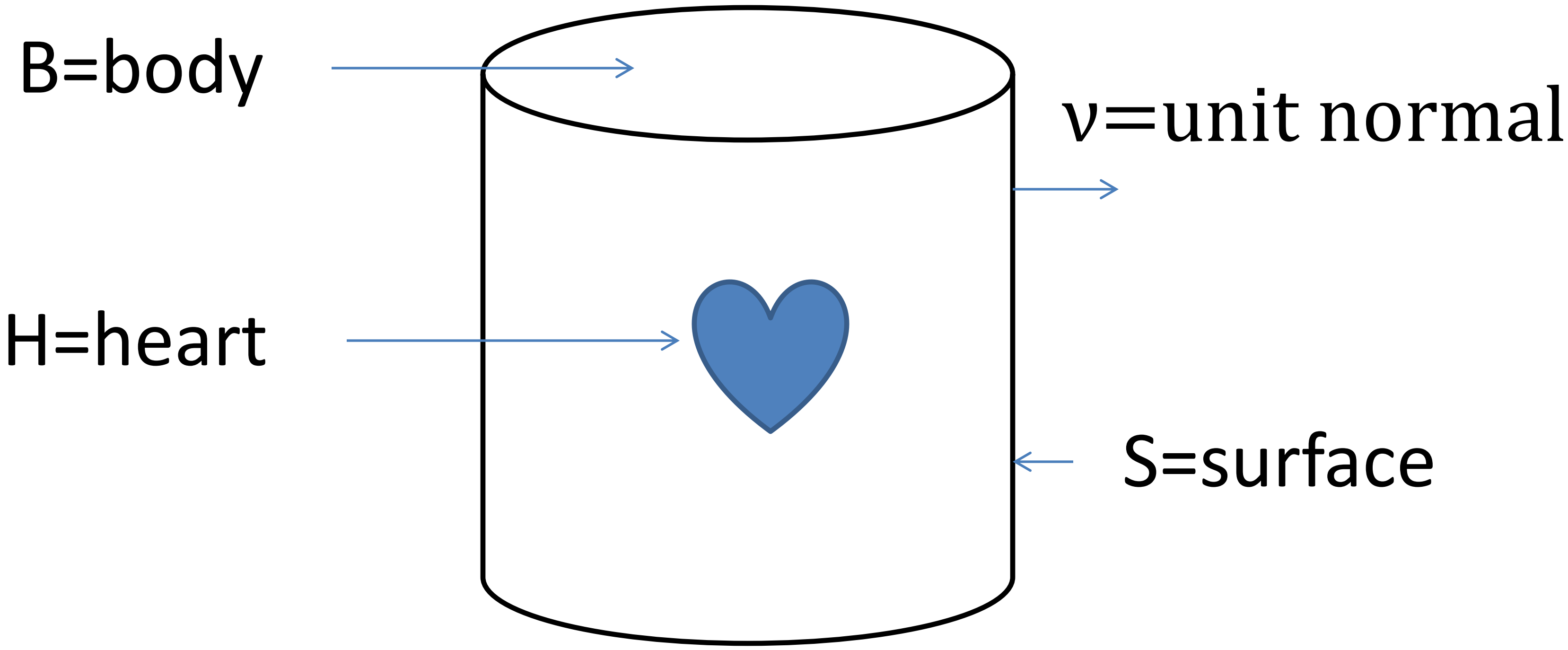
QRS interval Nonexistent

Interpretation Ventricular fibrillation

How does the heart produce the voltages on the bodies surface?

Forward problem.

The Standard Model



$$J(x,t) = \text{Total current density vector.}$$

$$\rho(x,t) = \text{charge density.}$$

$$E(x,t) = \text{Electric Field}$$

$$B(x,t) = \text{Magnetic Field}$$

$$\sigma(x,t) = \text{Conductivity}$$

$$J^{H}(x,t) = \text{Current density of sources in heart.}$$

$$J^{O}(x,t) = \text{Ohmic current density vector.}$$

Static approximation ; $\partial/\partial t = 0$.

Conservation of Charge;

$$\nabla \cdot J = -\partial\rho / \partial t \quad (= 0).$$

Ohm's law;

$$\mathrm{J} = \mathrm{J}^{\mathrm{O}} + \mathrm{J}^{\mathrm{H}}, \qquad \mathrm{J}^{\mathrm{O}} = \sigma\, \mathrm{E}$$

Farady's law;

$$\nabla \wedge \mathrm{E} = -\,\partial \mathrm{B}/\partial t \,(= 0).$$

$$\nabla \wedge \mathrm{E} = -\partial \mathrm{B}/\partial \mathrm{t} = 0 \Rightarrow$$

$$\mathrm{E} = -\nabla \mathrm{U}$$

$$\mathrm{U} = \text{Voltage or electrical potential.} \Rightarrow$$

$$\nabla \cdot \mathrm{J}^{\mathrm{O}} = \nabla \cdot \sigma \mathrm{E} = -\nabla \cdot \sigma \nabla U = -\nabla \cdot \mathrm{J}^{\mathrm{H}}$$

Standard Forward Model

$$\nabla \cdot \sigma \nabla U = \nabla \cdot \mathrm{J}^{\mathrm{H}}, \quad \text{in B}$$

$$\sigma \partial \mathrm{U} / \partial \nu = 0, \qquad \text{on S.}$$

$$\text{Given } \sigma \text{ and } \mathrm{J}^{\mathrm{H}}, \text{ find}$$

$$\mathrm{V} \equiv \mathrm{U}, \quad \text{on S.}$$

Since 1887 we've measured

$V(x,t)=U(x,t)$

on the chest S.

Forward problem:
given J^H and σ, find V.

Inverse (physician's) problem:
given V, find J^H (and σ).

Warning – not unique!

$$\mathrm{J}^{\mathrm{H}} \to \mathrm{J}^{\mathrm{H}} + \nabla \wedge F$$

How to find clinically useful solutions to the inverse problem?

Sylvester's solution;

Simulate ECGs by solving many forward problems with special J's and σ's.

Analog Computer Model of the Vectorcardiogram

By Ronald H. Selvester, M.D., Clarence R. Collier, M.D., and Robert B. Pearson, M.D.

THE formation of a model is an important phase of all scientific thinking in that it serves to demonstrate or explain the workings of an inherently complex natural system in terms that can be more readily understood.

In electrocardiography, the Einthoven triangle[1] has been used for years as a relatively simple geometric model of an essentially complex volume conductor, the human torso.

From the Medical Science Service, Rancho Los Amigos Hospital, Downey, California.

This model, in spite of many oversimplifications, has served for over 50 years as a useful reference frame for a vast amount of data and theory. More recently, the definitive mapping study by Scher[2] of the sequence of myocardial depolarization in dogs, again a simple model of a very complex phenomenon, has served to illuminate and clarify a vast body of empirical data. This model has added greatly to our understanding of previously confusing findings in electrocardiography and vectorcardiography.

The introduction of computer technics

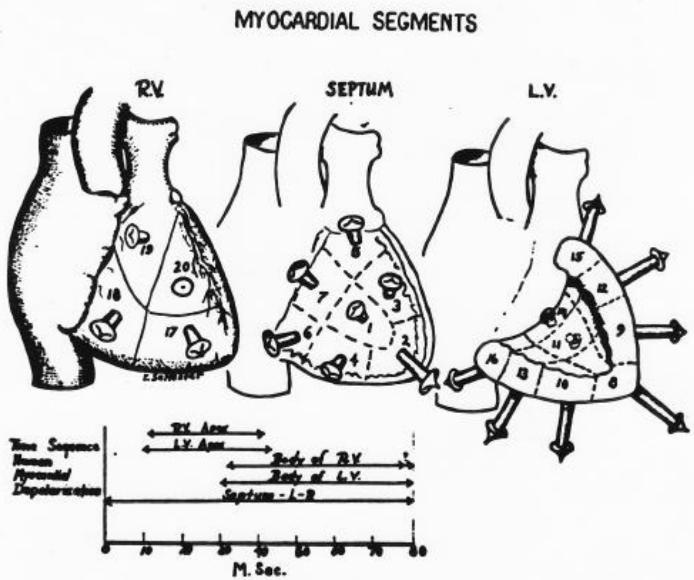
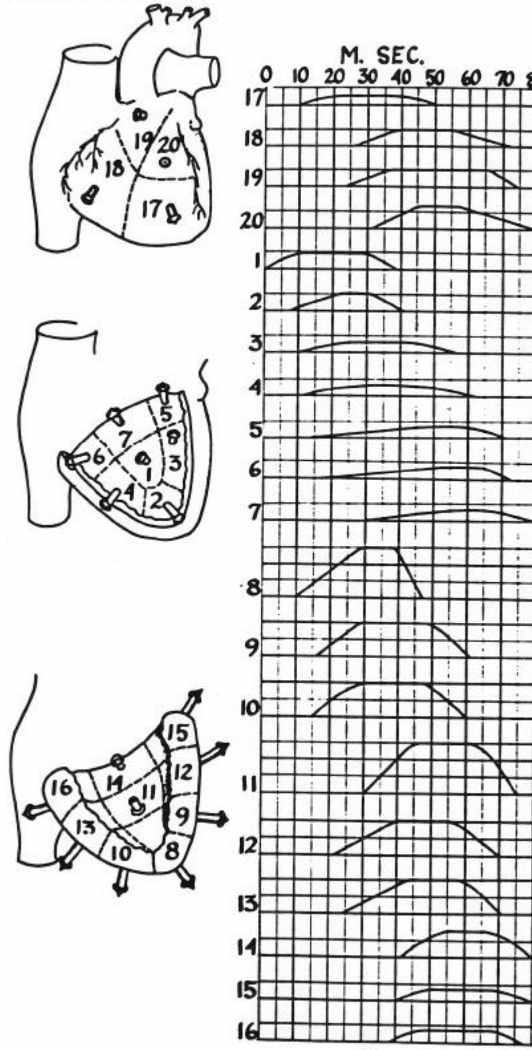


Figure 1

In the simulation the heart was pictured as consisting of 20 segments; this diagram gives an approximation of the direction assigned to the vector representing each segment.



Segment	l	m	n
17	+.416	−.618	+.666
18	−.433	−.498	+.750
19	−.829	0	+.559
20	0	0	+1.000
1	−.407	+.704	+.581
2	+.634	−.634	+.444
3	0	+.342	+.940
4	−.498	−.498	+.712
5	−.616	+.287	+.734
6	−.844	−.488	+.227
7	−.811	+.143	+.568
8	+.495	−.495	−.286
9	+.983	−.086	−.1735
10	+.259	−.968	−.0457
11	+.706	−.0618	−.706
12	+.700	+.588	−.404
13	−.472	−.818	−.330
14	−.259	+.0226	−.965
15	+.253	+.946	−.213
16	−.750	−.630	−.202

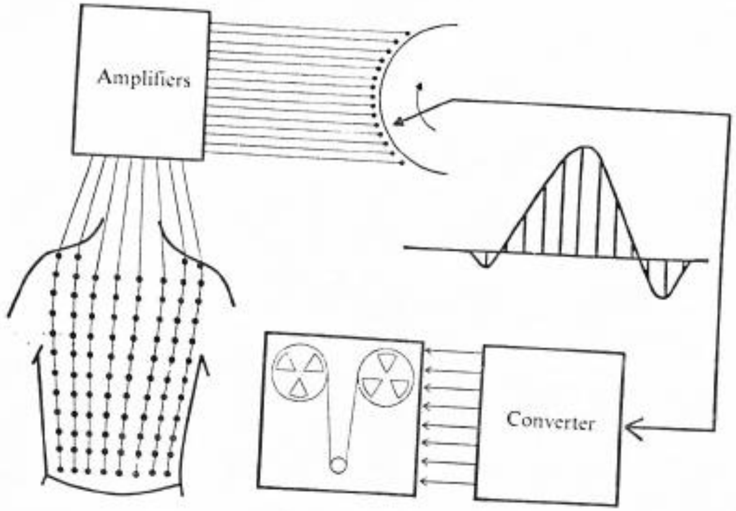
Figure 3

The time history of the current field strength from each of the 20 segments is shown in the central portion of this figure. To the right are the direction cosines (L, M, and N) for each of

Can we do better?

Body Surface mapping

1963 – Taccardi
1978 – Colli-Franzone



(a)

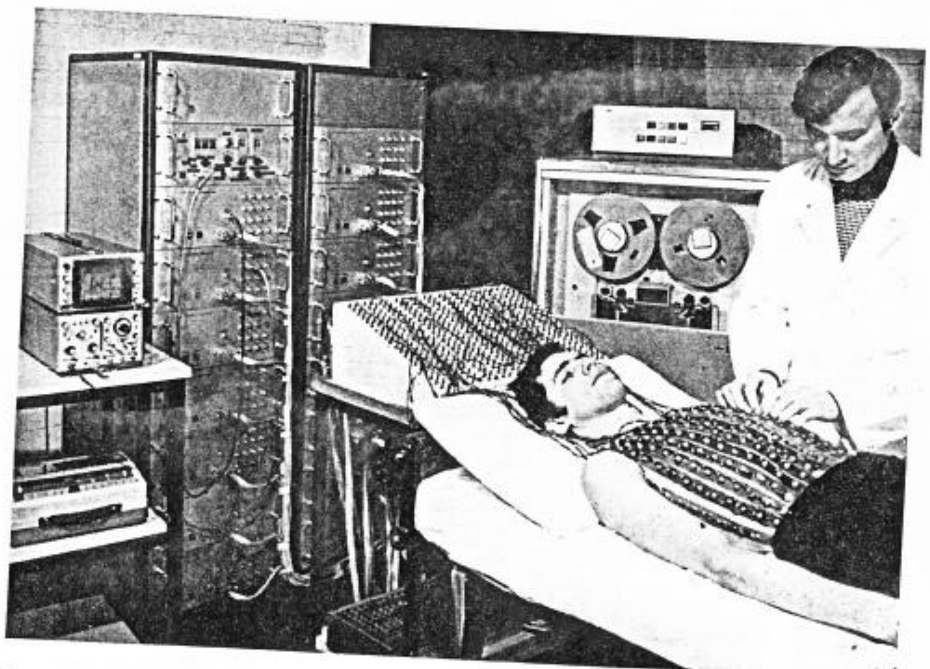


FIG. 19.7. (a) Block diagram of the 240-channel instrument, illustrating the electrodes, amplifiers, multiplexer, analog-to-digital converter and digital tape-recorder. (b) General view of the patient, input connectors, amplifiers, control unit (*upper left panel*), monitoring oscilloscope, and IBM 2401/5 digital tape unit.

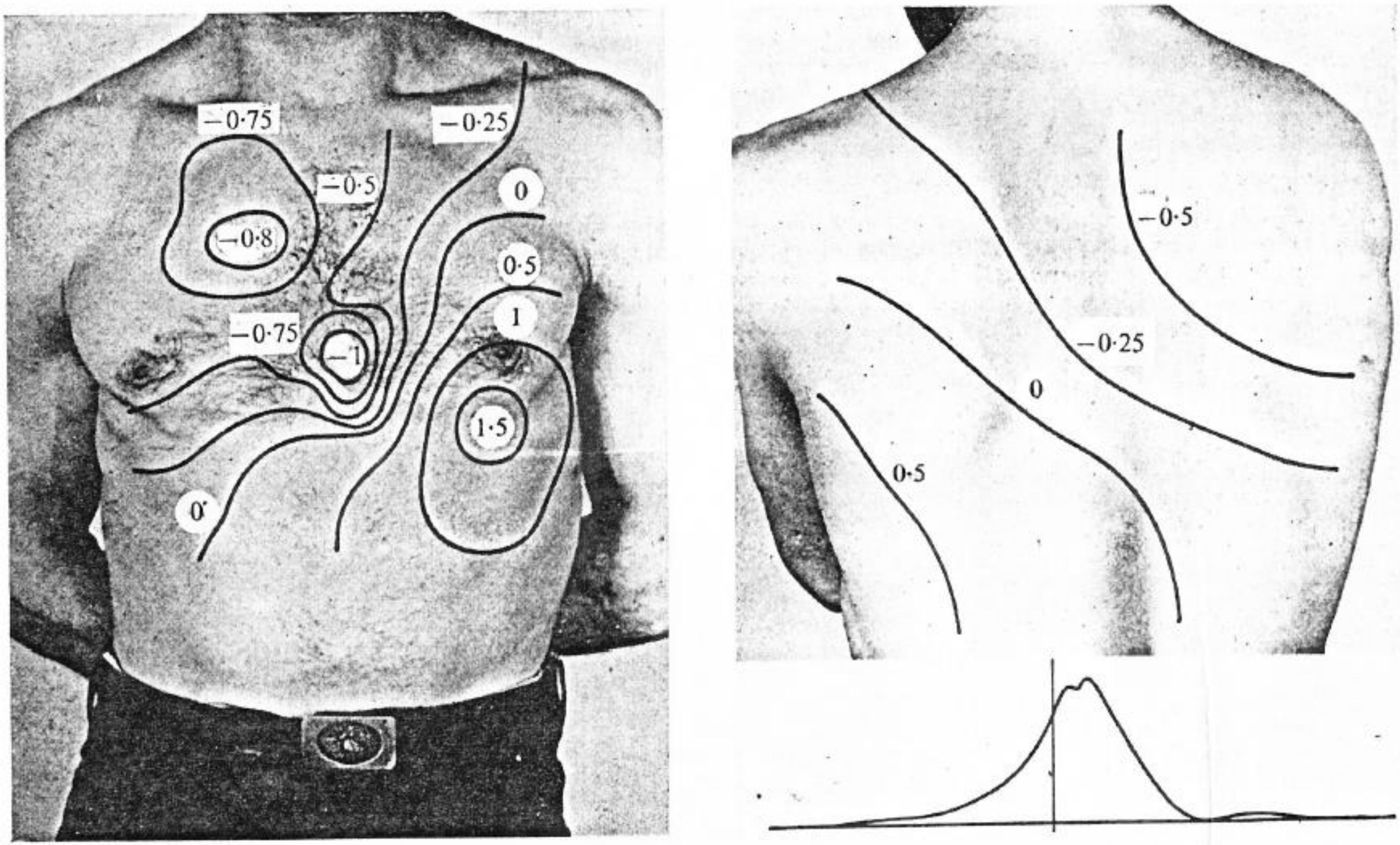


FIG. 19.4. Distribution of equipotential lines on the thoracic surface of a normal human subject at the instant of time indicated by the vertical line intersecting the enlarged QRS complex at the lower right of the figure. Two separate minima are present. (After Taccardi (1963).)

Colli-Franzone

Reconstruct epicardial potentials v

from body surface map V, i.e. given;

$\nabla \cdot \sigma \nabla U = 0$, between ∂H and S.

$U = V$, and $\sigma \partial U / \partial \nu = 0$, on S.

Find $v = U$, on ∂H.

cardio » e 156 sulla superficie laterale del bagno. La geometria del bagno dell'esperimento e la locazione degli elettrodi sono state ricostruite da fotografie riprese dopo l'esperimento. Va osservato che questa geometria non è concettualmente differente dalla geometria toracica se non per l'omogeneità del mezzo conduttore e la distanza media molto più grande esistente fra l'epicardio e la superficie laterale del bagno (fig. 1).

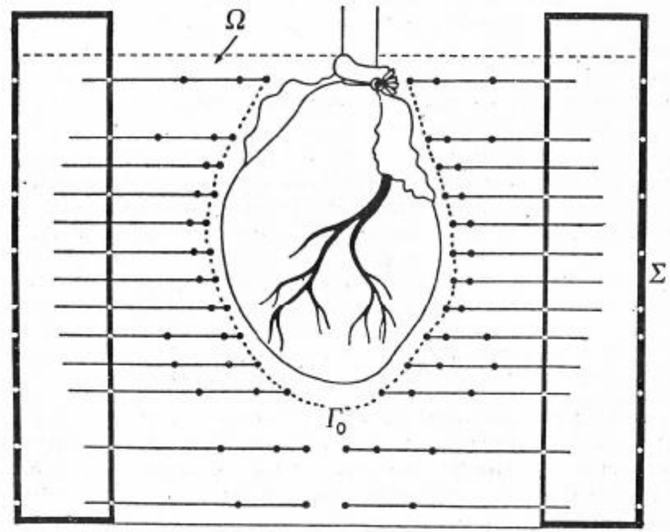


Fig. 1 Illustrazione schematica del bagno sperimentale contenente il cuore perfuso.

Indicata con E_{sp} (E^*_{sp}) la mappa epicardica (ad 1 cm dall'epicardio) sperimentale consideriamo il seguente problema « diretto »:

$$(16) \qquad \Delta V = 0 \text{ in } \Omega\,, \quad V = u \text{ su } \Gamma_0\,, \quad \frac{\partial V}{\partial n} = 0 \text{ su } \Gamma_1$$

con $u = E_{sp}$ (e E^*_{sp}); indicheremo allora con S_{cal} (o S^*_{cal}) $= V|_\Sigma$ la mappa di superficie simultata risolvendo numericamente (16), con S_{sp} quella sperimentale e con E_{cal} la mappa epicardica ricostruita risolvendo il problema (15) con $z_d = S_{cal}$ o $z_d = S_{sp}$ (4).

(4) Notiamo che S_{cal} è dunque affetta da errori dovuti alla discretizzazione di (16).

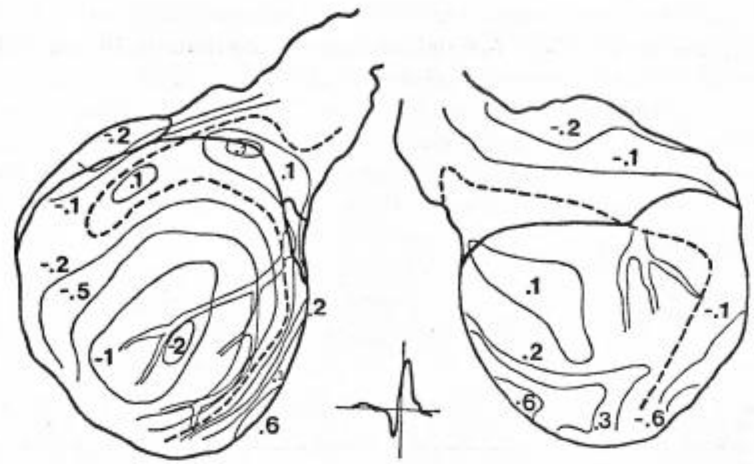


Fig. 3 Distribuzione di potenziale epicardico (mV) ricostruita a 30 ms dall'inizio del QRS (E_{cal} a 30 ms da S_{cal}).

l'errore quadratico relativo alla mappa epicardica sperimentale

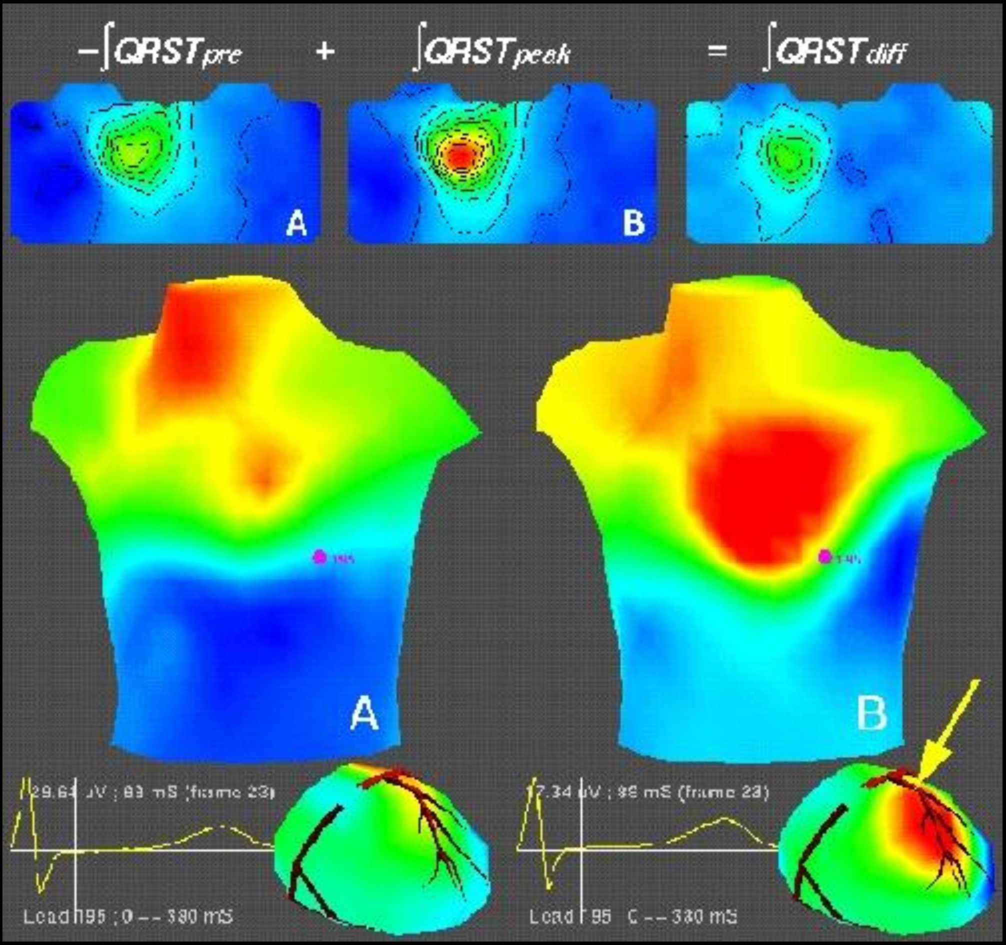
$$ERR = \left(\left(\sum_{i=1}^{122} |E^i_{\mathrm{cal}} - E^i_{\mathrm{sp}}|^2 \right) \Big/ \sum_{i=1}^{122} |E^i_{\mathrm{sp}}|^2 \right)^{\frac{1}{2}}$$

delle stime della distribuzione epicardica ottenute a partire da S_{cal} a 30 ms. La miglior stima sia qualitativa che quantitativa è ottenuta con il reticolo R_3 (fig. 2-3).

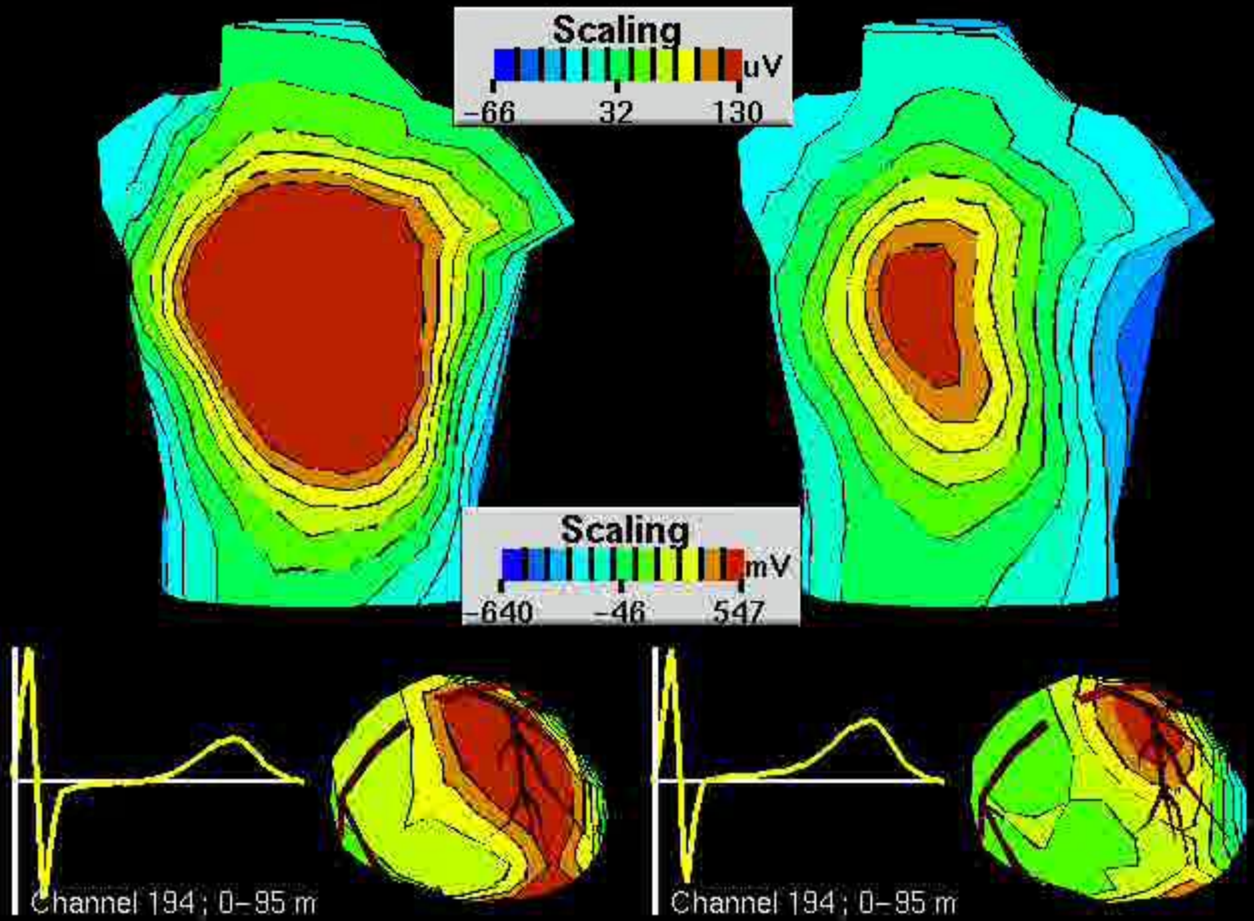
Tavola 2

t	z_d	u	ERR	ε
30 ms	S_{cal}	E_{sp}	0.371	$0.1 \cdot 10^{-7}$
30 ms	S^*_{cal}	E^*_{sp}	0.218	$0.1 \cdot 10^{-6}$
30 ms	$S^*_{\mathrm{cal}} \pm 14\ \mu\mathrm{V}$	E^*_{sp}	0.333	$0.2 \cdot 10^{-6}$
10 ms	S_{cal}	E_{sp}	0.222	$0.3 \cdot 10^{-9}$

R. Macleod



$-\int QRST_{pre} + \int QRST_{peak} = \int QRST_{diff}$
A
B
A
B
Lead 195 ; 0 -- 380 mS
C -- 380 mS
(frame 23)


Scaling
uV
-66
32
130
Scaling
mV
-640
-46
547
Channel 194 ; 0-95 m
Channel 194 ; 0-95 m

Problems :

1. How to find conductivity σ?

Goes back to Schlumberger – 1912

2. How to get pumping information?

Electrical Impedance Tomography and Spectroscopy

David Isaacson

Jonathan Newell

Gary Saulnier

RPI

With help from

D.G.Gisser, M.Cheney
J.Mueller,S.Siltanen

and

Denise Angwin, B.S. Greg Metzger, B.S . Hiro Sekiya, B.S. _Steve Simske, M.S. Kuo-Sheng Cheng, M.S. Luiz Felipe Fuks, Ph.D. Adam Stewart Andrew Ng, B.S. Frederick Wicklin, M.S. Scott Beaupre, B.S. Andrew Kalukin, B.S. Tony Chan, B.S. Matt Uyttendaele, M.S. Steve Renner, M.S. Laurie Christian, B.S. Van Frangopoulos, M.S. Tim Gallagher, B.S. Lewis Leung, B.S. Jeff Amundson, B.S. Kathleen Daube, B.S. Candace Meindl Matt Fisher Audrey Dima, M.Eng. Skip Lentz Nelson Sanchez, M.S. Clark Hochgraf John Manchester Erkki Somersalo, Ph.D. Hung Chung Molly Hislop Steve Vaughan Joyce Aycock Laurie Carlyle, M.S. Paul Anderson, M.S. John Goble, Ph.D. Dan Kacher Chris Newton, M.S. Brian Gery Qi Li Ray Cook, Ph.D. Paul Casalmir Dan Zeitz, B.S. Kris Kusche, M.S. Carlos Soledade, B.S. Daneen Frazier Leah Platenik Xiaodan Ren, M.S. David Ng, Ph. D. Brendan Doerstling, Ph.D. Mike Danyleiko, B.S. Cathy Caldwell, Ph.D. Nasriah Zakaria Peter M. Edic, Ph. D. Bhuvanesh Abrol Julie Andreasson, B.S. Jim Kennedy, B.S. Trisha Hayes, B.S. Seema Katakkar Yi Peng Elias Jonsson, Ph. D. Pat Tirino M.S. Hemant Jain, Ph. D. Rusty Blue, Ph. D. Julie Larson-Wiseman, Ph. D.

Impedance Imaging Problem;

How can one make clinically useful images of the electrical conductivity and permittivity inside a body from measurements on a body's surface?

Potential Applications

I. Continuous Real Time Monitoring of Function of:

1. **Heart**
2. **Lung**
3. Brain
4. Stomach
5. Temperature

II. Screening:

1. **Breast Cancer**
2. Prostate Cancer

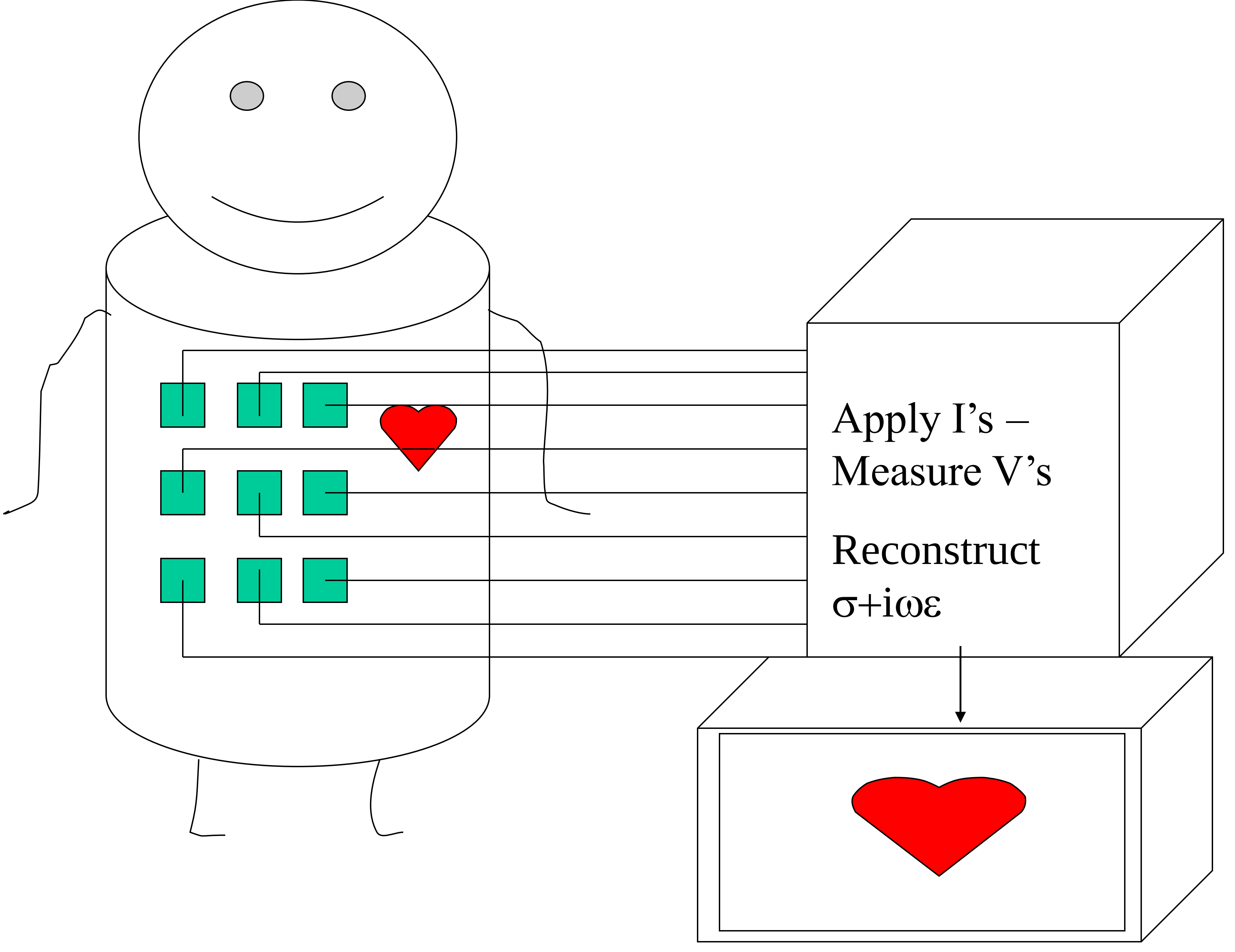
III. Electrophysiological Data for Inverse problems in:

1. **EKG**
2. EEG
3. EMG

Reasons

TISSUE	Conductivity S/M	Resistivity Ohm-Cm
Blood	.67	150
Cardiac Muscle	.2	500
Lung	.05	2000
Normal Breast	.03	3000
Breast Carcinoma	.2	500

Procedure
For Imaging Heart and Lung Function in 3D
<u>Electrical Impedance Tomography</u>


Apply I's –
Measure V's
Reconstruct
σ+iωε

Apply current density j;

$$\nabla \cdot \sigma \nabla W = \nabla \cdot \mathrm{J}^{\mathrm{H}}, \quad \text{in B}$$

$$\sigma \partial \mathrm{W}/\partial \nu = \mathrm{j}, \qquad \text{on S.}$$

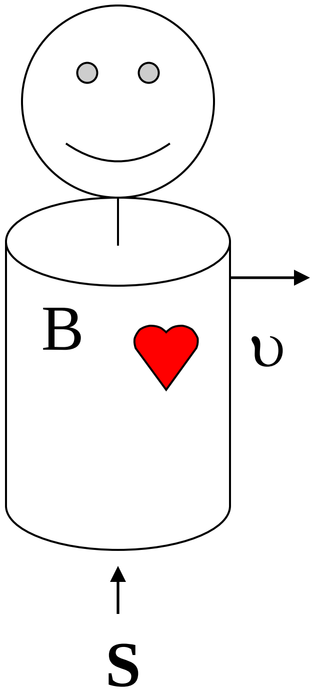
$$\mathrm{J}^{\mathrm{H}}(x,\omega) \approx 0, \text{ for } \omega/2\pi > 100\,\mathrm{Hz}.$$

$$\mathrm{j}(\mathrm{x,t}) \equiv \mathrm{j(x)e}^{\mathrm{i}\omega \mathrm{t}}, \text{ for } \omega/2\pi > 100\,\mathrm{Hz}.$$

$$\mathrm{W(x,t)} = U(\mathrm{x})\mathrm{e}^{\mathrm{i}\omega \mathrm{t}} + U^{H}(x,t)$$

Main Equation

$$\nabla \cdot \sigma \nabla U = 0$$



$$\nabla \cdot \sigma \nabla U = 0 \quad \text{in B}$$

$$\sigma \partial U / \partial \nu = j \quad \text{on S}$$

Forward Problem:

Given conductivity **σ** and current density **j** find v = U on S.

i.e.

Find the Neuman to Dirichlet map:

$$\mathbf{R}(\sigma)\mathbf{j} = \mathbf{v}.$$

Where

$$\mathbf{R}(\sigma): \mathbf{H}^{-1/2}(\mathbf{S}) \rightarrow \mathbf{H}^{+1/2}(\mathbf{S})$$

Inverse Problem:
Given

$\mathbf{R}(\boldsymbol{\sigma})$

Find

$\boldsymbol{\sigma}$

Does it have unique solution?

Yes!

Langer – 1934

Calderon

Kohn and Vogelius

Sylvester and **Uhlmann**

Nachman

Astala and Paivarinta

…

4. How can we reconstruct useful images?

1. Linearization (Noser 2-D, Toddler 3-D); Fast, useful, not as accurate for large contrast conductivities.
2. Optimization (Regularized Gauss-Newton);

   Slow, more accurate , iterative methods.
3. Direct methods (Layer stripping, Complex Geometrical Optics, D-Bar);

   Solve full non-linear problem, no iteration!

What can a linearization do?

Noser – a 2-D reconstruction

Toddler – a 3-D reconstruction

(both assume conductivity differs only a little from a constant.)

FNoser - Fast ,20 frames/sec

Real time imaging of Cardiac and Lung function shown in the following examples.

Linearizations
NOSER (S.Simske,…)
FNOSER(P.Edic,…)
TODDLER(R.Blue,…)

$\nabla\cdot\sigma\nabla u^m = 0$	$\nabla\cdot\sigma_0\nabla u_0^n = 0$
$\sigma\,\partial u^m/\partial\nu = j^m$	$\sigma_0\,\partial u_0^n/\partial\nu = j_0^n$

$$u_0^n\nabla\cdot\sigma\nabla u^m = 0 \qquad u^m\nabla\cdot\sigma_0\nabla u_0^n = 0$$

$$\int u_0^n\nabla\cdot\sigma\nabla u^m - u^m\nabla\cdot\sigma_0\nabla u_0^n\,dx = 0$$

$$\Downarrow$$

$$\int_S u_0^n\sigma\,\partial_\nu u^m - u^m\sigma_0\partial_\nu u_0^n\,dS = \int_B (\sigma-\sigma_0)\nabla u^m\cdot\nabla u_0^n\,dx$$

$$\int_S u_0^n \sigma \partial_\nu u^m - u^m \sigma_0 \partial_\nu u_0^n \, dS = \int_S u_0^n j^m - u^m j^n \, dS =$$

$$< j^m, (R(\sigma) - R(\sigma_0)) j^n > =$$

$$\text{Data(n,m)} =$$

$$\int_B (\sigma - \sigma_0) \nabla u^m \cdot \nabla u_0^n \, dx$$

$$\text{If } \delta\sigma \equiv \sigma - \sigma_0 << \sigma_0 \text{ then } u^m = u_0^m + O(\delta\sigma)$$

$$Data(n,m) = \int_B (\sigma - \sigma_0) \nabla u^m \cdot \nabla u_0^n \, dx$$

$$= \int_B \delta\sigma \nabla u_0^m \cdot \nabla u_0^n \, dx + O(\delta\sigma^2)$$

$$Data(n,m) \approx \int_B \delta\sigma \, \nabla u_0^m \cdot \nabla u_0^n \, dx$$

$$\text{Choose BASIS}, \{\psi_k(x)\},$$

$$\delta\sigma(x) = \sum_{\mathrm{k}} C_k \psi_k(x)$$

$$\text{Thus only need to solve;}$$

$$Data(m,n) = \sum_{\mathrm{k}} C_k \int_B \psi_k(x) \, \nabla u_0^m \cdot \nabla u_0^n \, dx$$

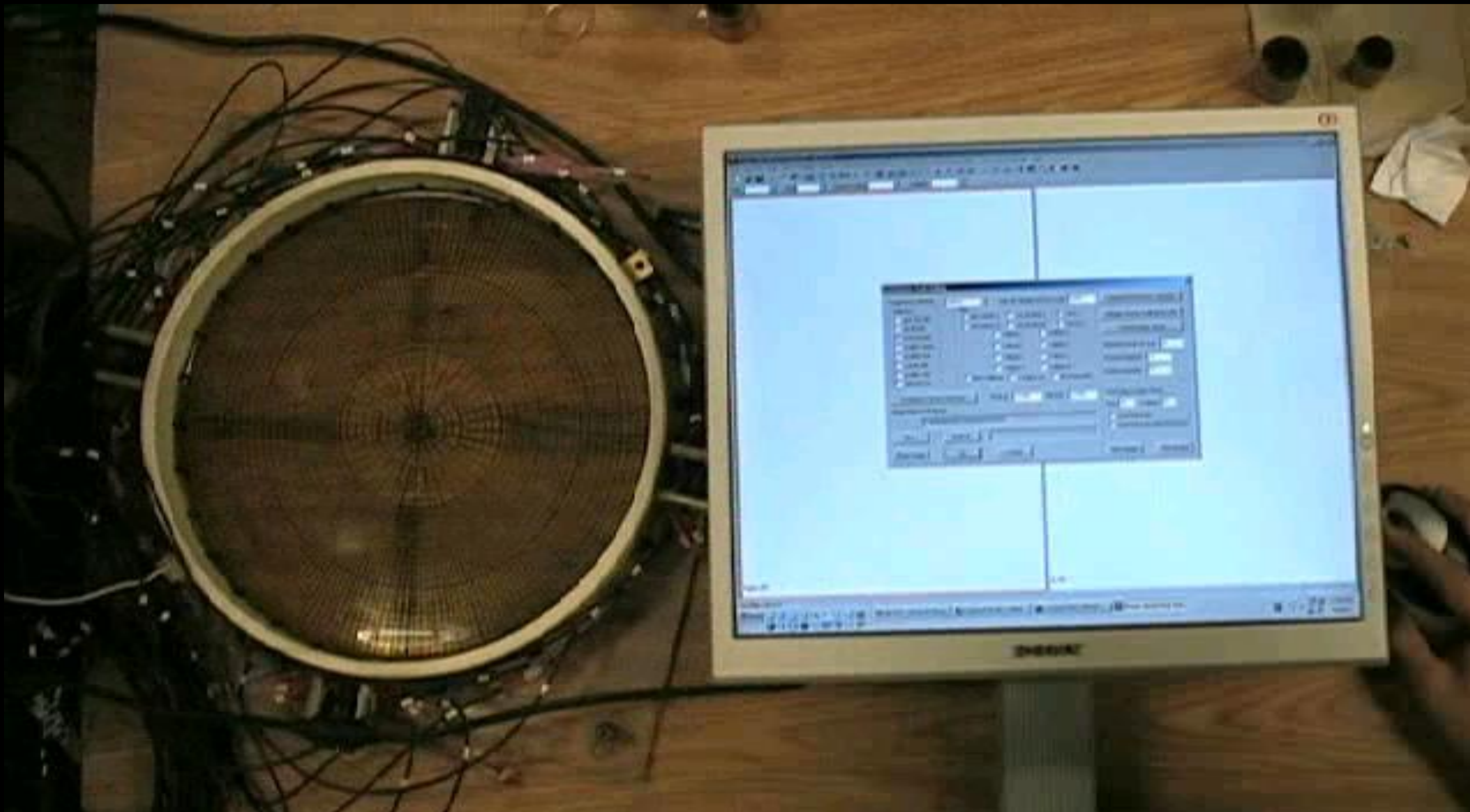
$$Data(m,n) = \sum_{\mathrm{k}} M_{m,n}^{k} C_k$$

Does it work?

Test by experiment

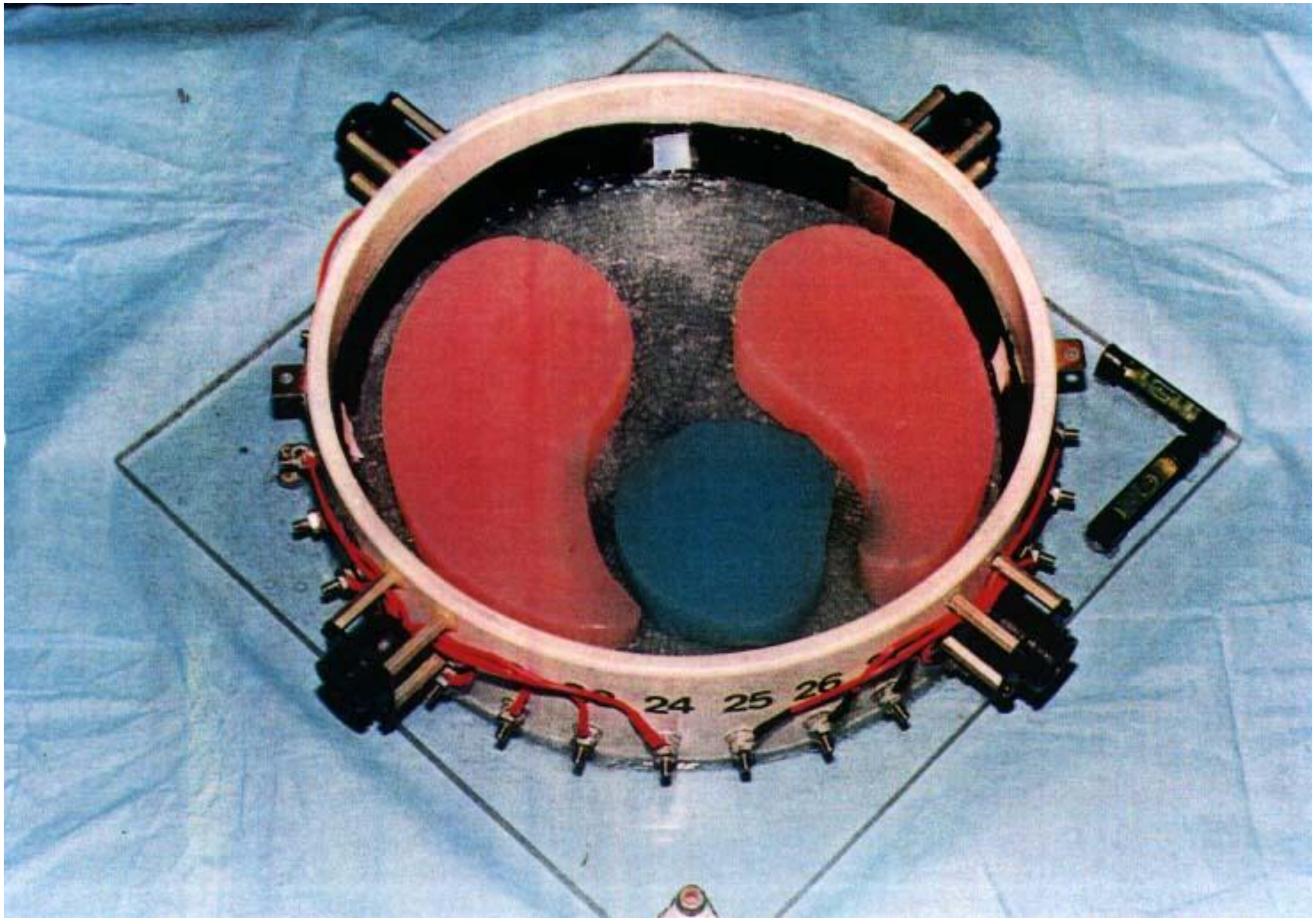
ACT 3

- 32 Current sources
- 32 Voltmeters
- 32 Electrodes
- 30 KHZ
- 20 Frames / Sec
- Accuracy > .01%

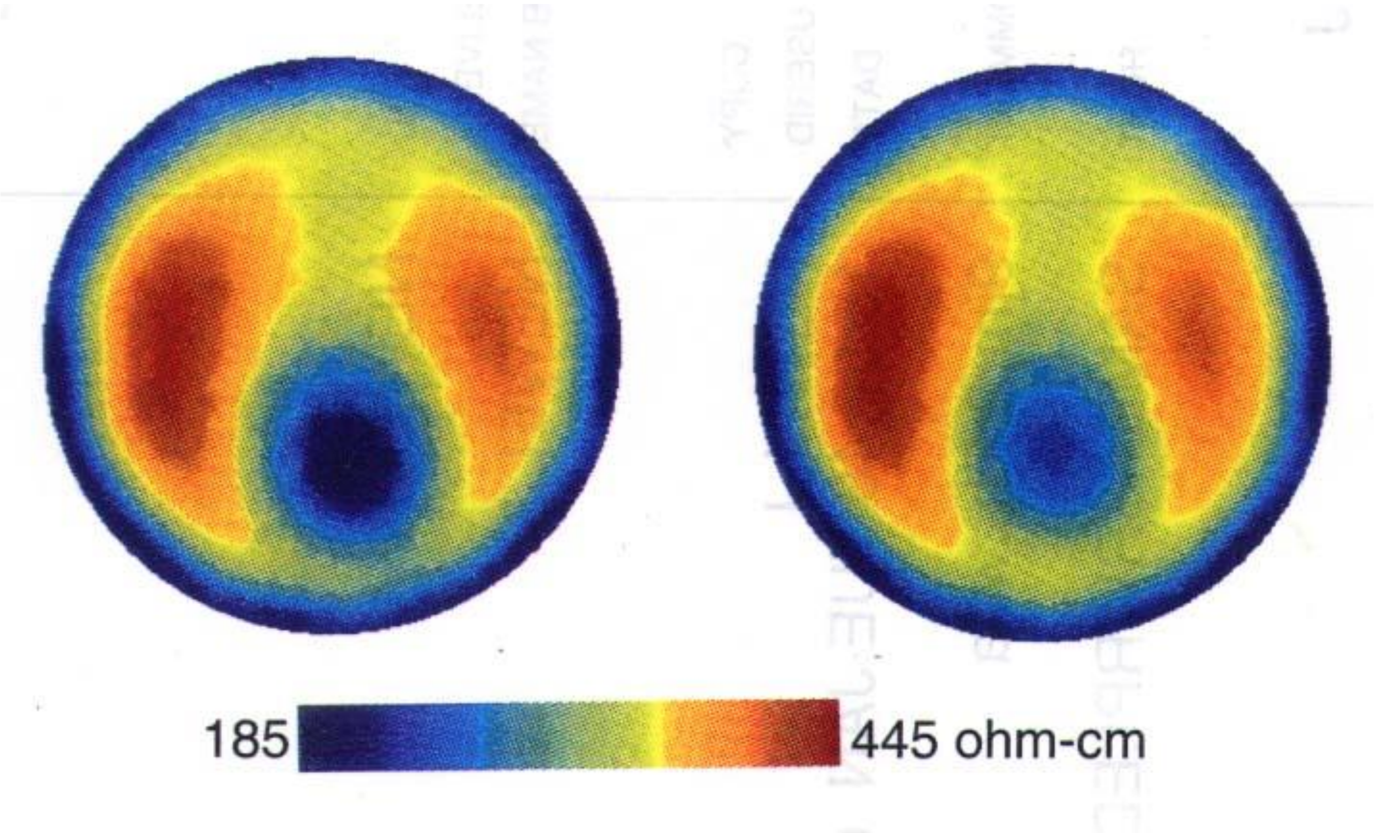


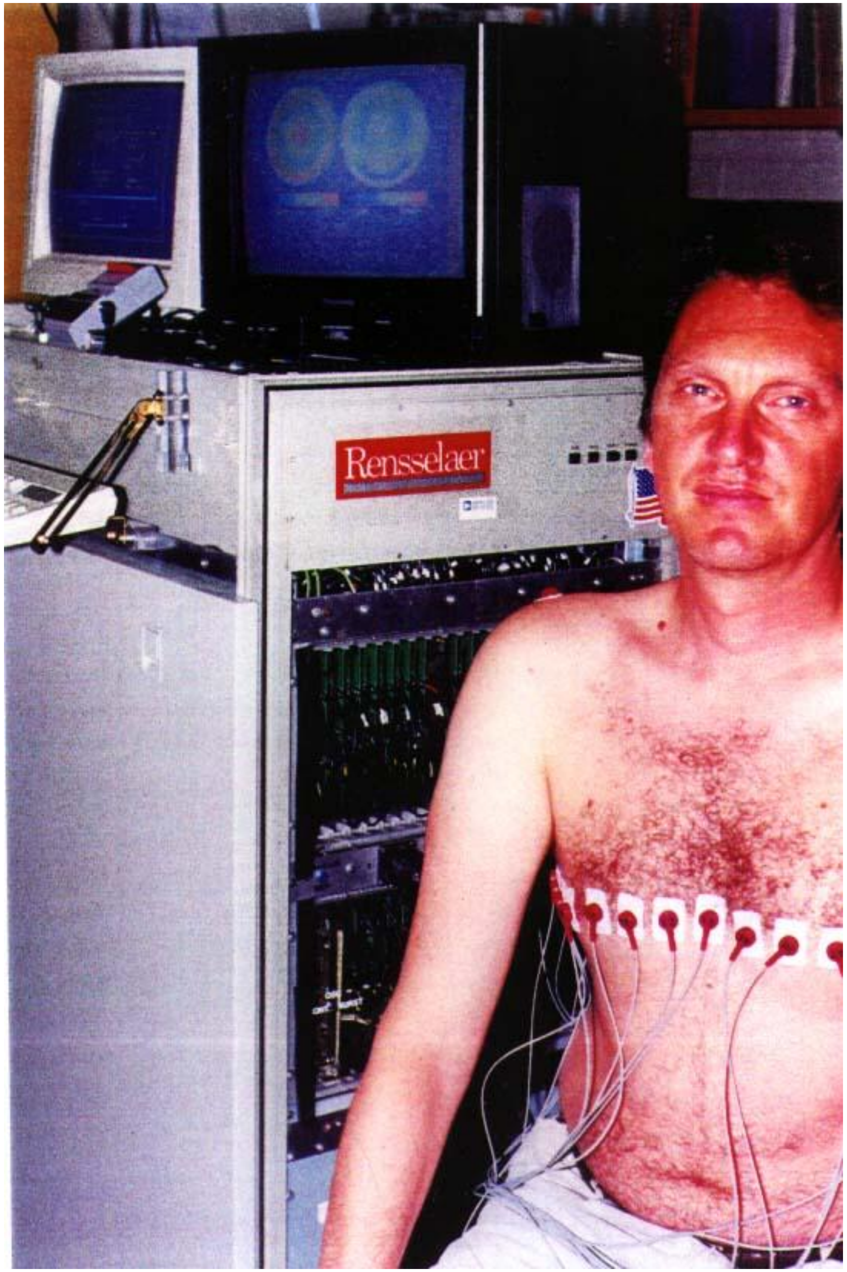
Can it image heart and lung function?

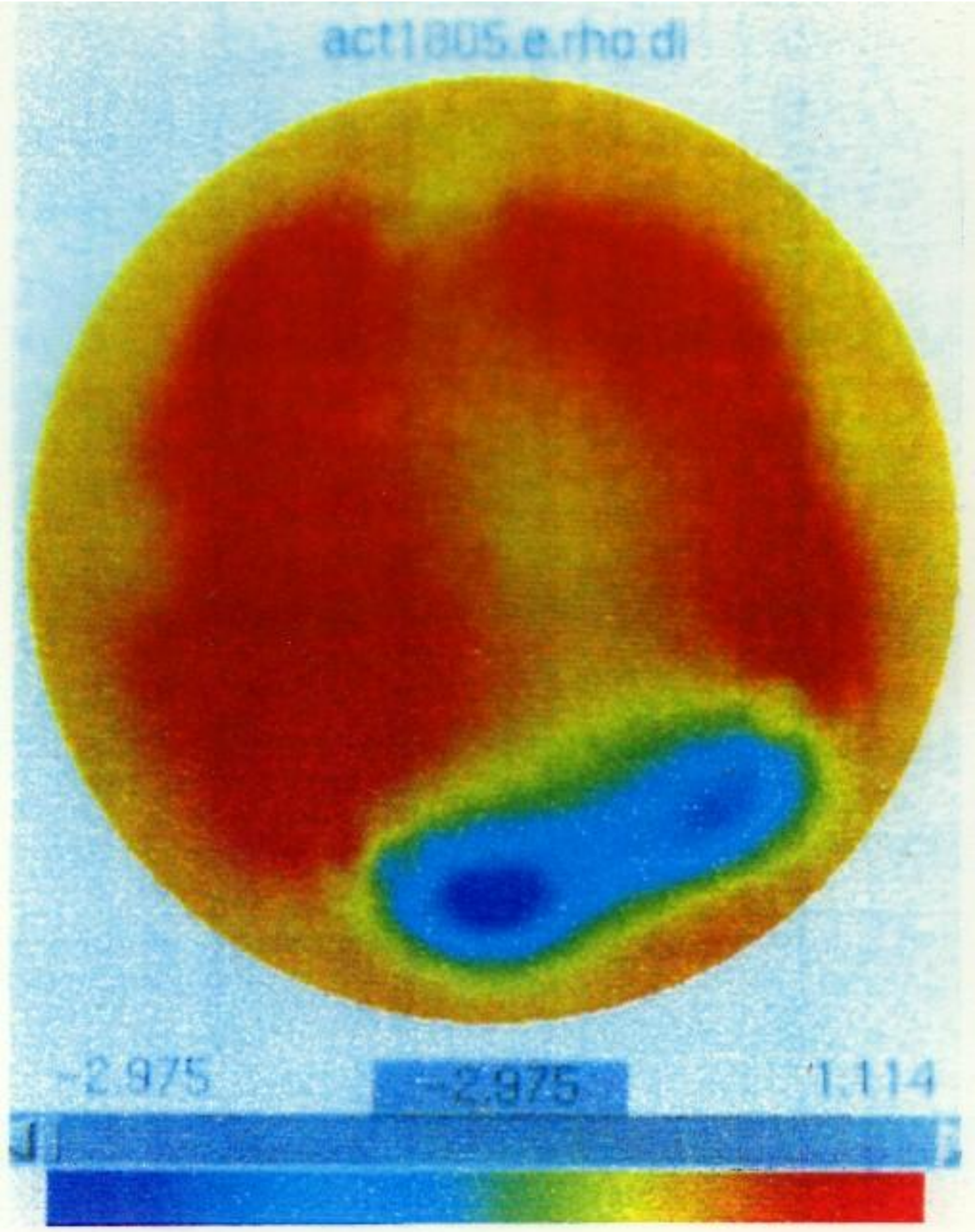
Phantom

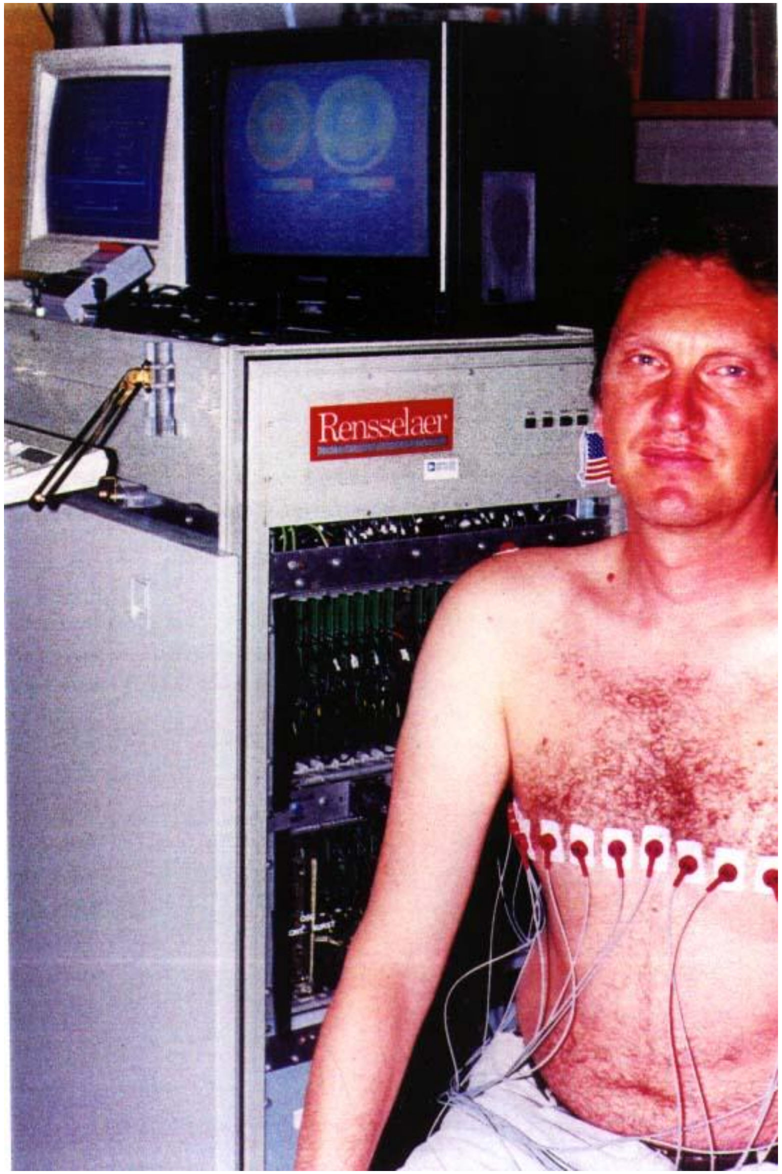


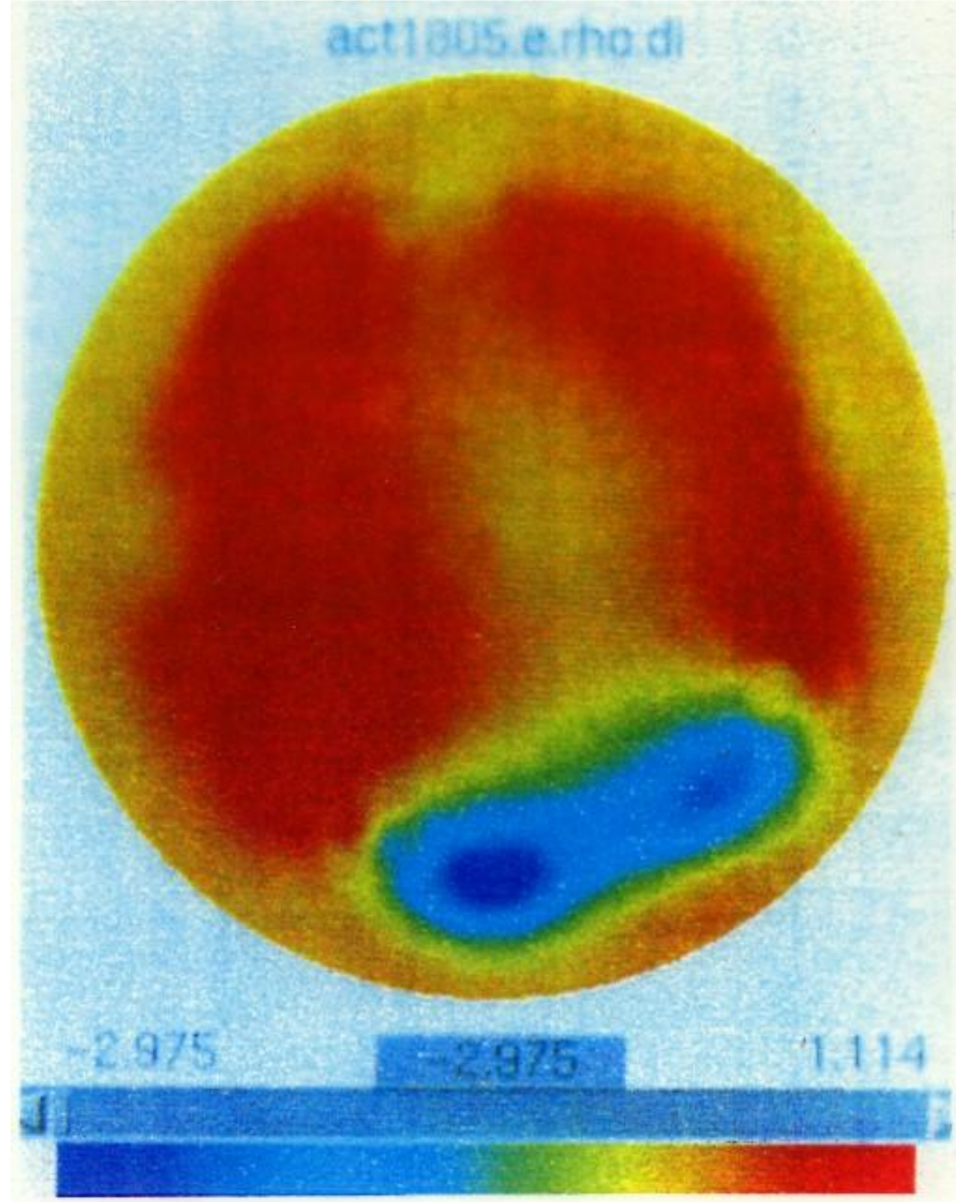
Reconstructions




Rensselaer


act1805.e.rho.di
-2.975
-2.975
1.114

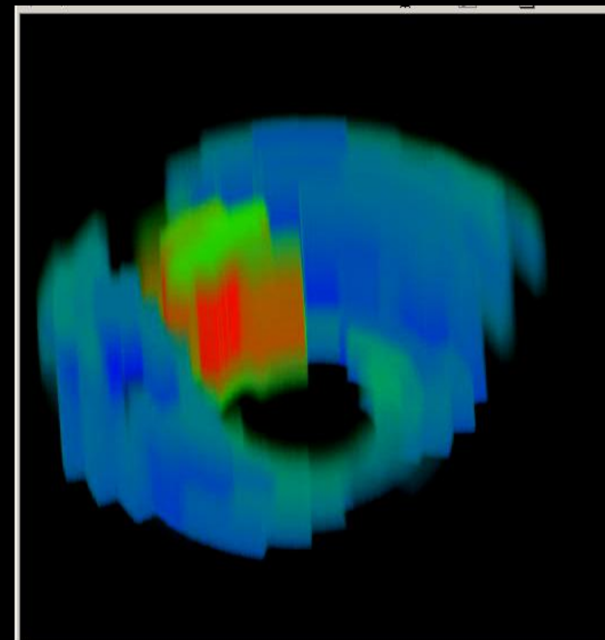


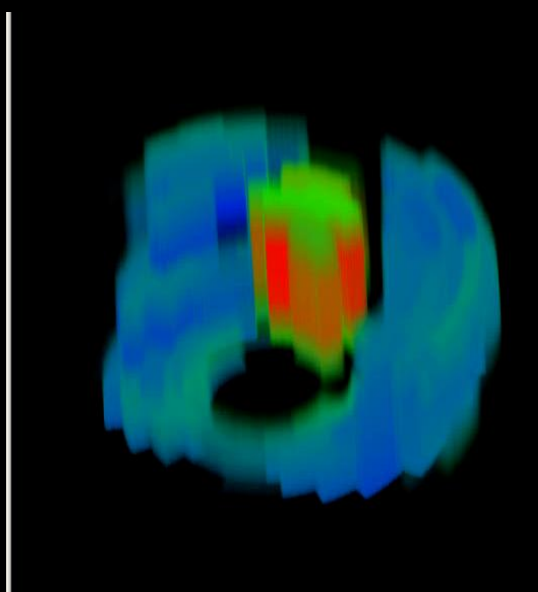


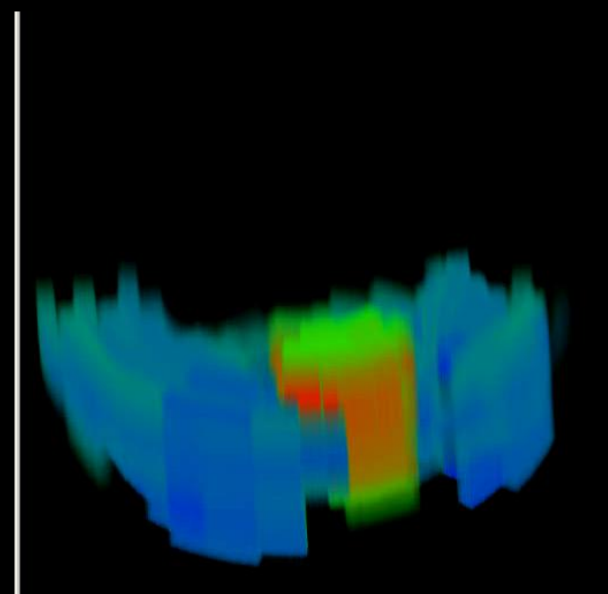
<u>ACT 3 imaging blood</u> as it leaves the <u>heart</u> (blue) and fills the <u>lungs</u> (red) during <u>systole</u>.

Show 2D Ventilation and Perfusion Movie

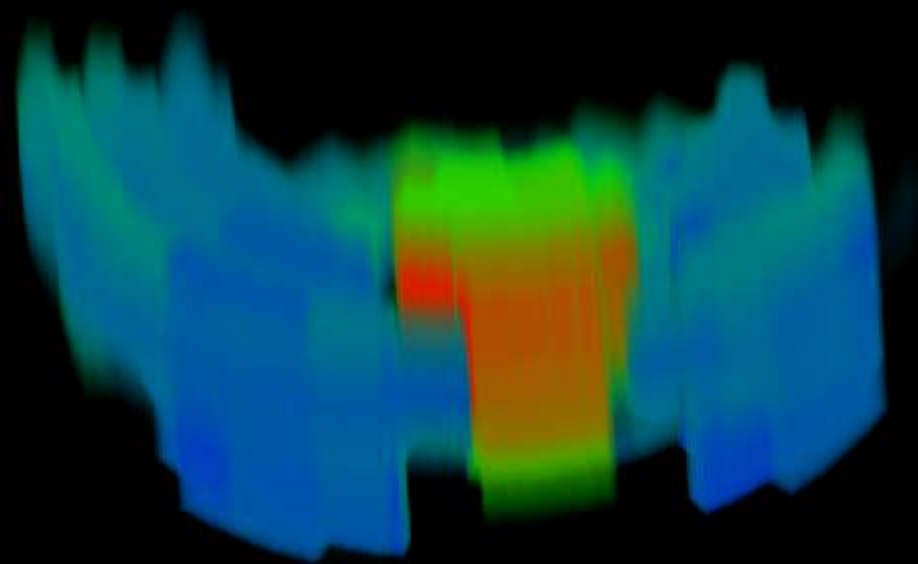
3D Electrode Placement





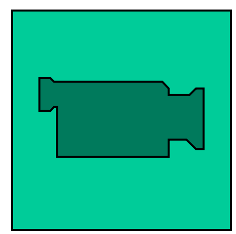


Heart Lung Static Image



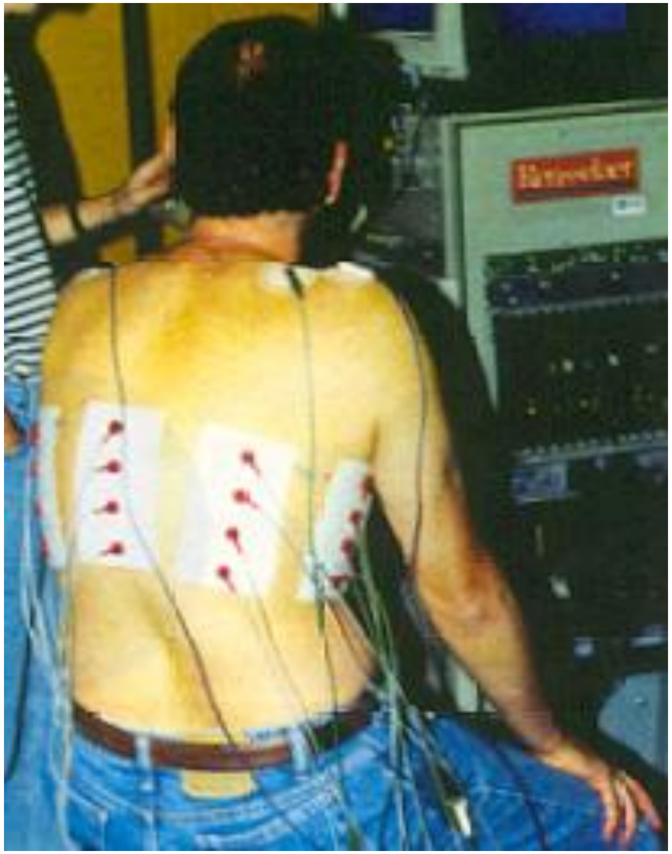
3D Heart lung

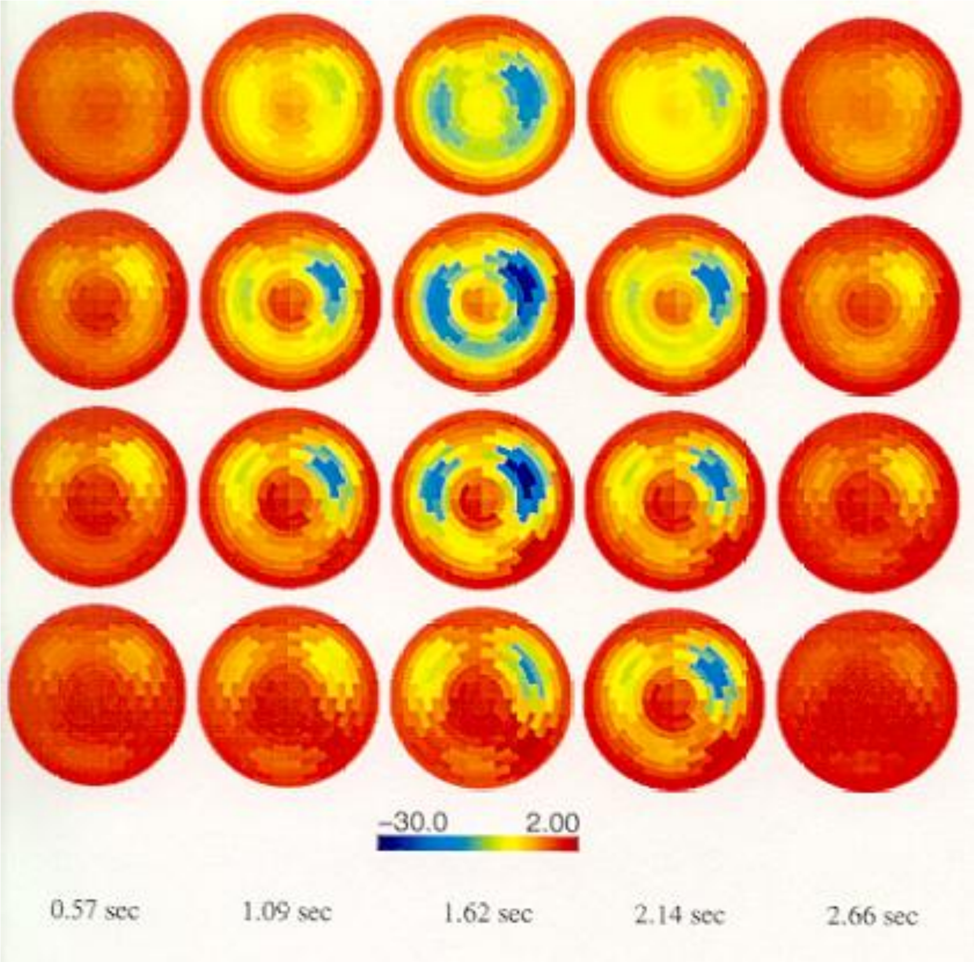
Show Heart Lung View from other source



Ventilation in 3D

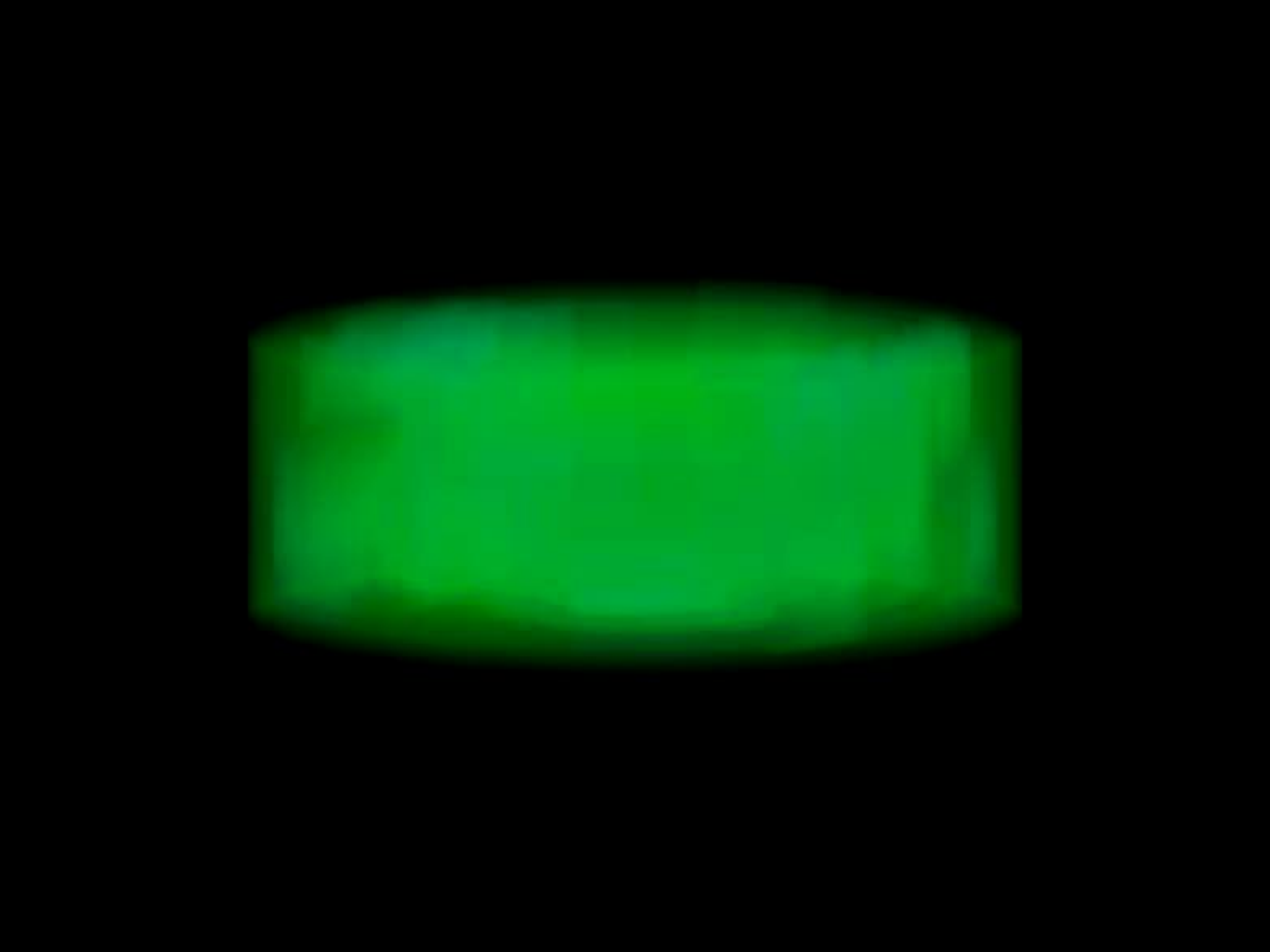
3D Human Results





- Images showing conductivity changes with respiration

Cardiac in 3D



How can one get more accurate values of the conductivity, less artifact,and still be fast?

Nachman's D-Bar method.
J. Mueller , S. Siltanen, D.I.

Special thanks to A. Nachman.

Nachman’s D-Bar method.

- Convert inverse conductivity problem to an Unphysical Inverse Scattering Problem for the Schrodinger Equation.
- Use the measured D-N map to solve a boundary integral equation for the boundary values of the exponentially growing Faddeev solutions .
- Compute the unphysical Scattering transform in the complex k-plane from these boundary values.
- Solve the D-Bar integral equation in the whole complex k-plane for the Faddeev solutions in the region of interest.
- Take the limit as k goes to 0 of these solutions to recover and display the conductivity in the region of interest.

Problem: Find the Conductivity σ from the measured Dirichlet to Neumann map Λ_σ

Assume:

$$\nabla \cdot \sigma \nabla u = 0 \qquad \text{inside B.}$$

$$\mathrm{u} = \mathrm{V} \qquad \text{on } \partial \mathrm{B}.$$

$$\Lambda_\sigma \mathrm{V} = \sigma\, \partial\, \mathrm{u} / \partial \nu \qquad \text{on } \partial \mathrm{B}.$$

$$\sigma = 1 \quad \text{in a neighborhood of } \partial \mathrm{B}.$$

Let ;

$$\Psi = \Psi(p,\zeta) \equiv \sigma^{1/2} u,$$

$$q = q(p) \equiv \sigma^{-1/2} \Delta \sigma^{1/2}$$

Then

$$-\Delta \Psi + q \Psi = 0 \qquad \text{in } B$$

$$\Lambda_\sigma \Psi = \partial\Psi/\partial\nu \qquad \text{on } \partial B$$

and $q = 0$ in a neighborhood of ∂B.

Look for Solutions Ψ on all of R^n $(n \geq 2)$ with

$q = 0$ outside ∂B that satisfy

$\Psi \approx \exp(i\zeta \cdot p)$ as $|p| \to \infty$, where $\zeta \cdot \zeta = 0$.

In R^2 take $\zeta = k(1,i)$ where $k = k_1 + ik_2$

Let

$$\Psi = \Psi(p,\zeta) = \exp(i\zeta \cdot p)\,\mu(p,\zeta)$$

where $\mu \to 1$ as $|p| \to \infty$.

Observe that

$$(-\Delta - 2i\zeta \cdot \nabla)\mu + q\mu = 0$$

and $\mu \to 1$ as $|p| \to \infty$.

We may recover σ from μ by the property that;

$$\sigma^{1/2}(p) = \mu(p,0) = \lim_{\zeta \to 0} \mu(p,\zeta).$$

Reason:

$$-\Delta\mu(p,0) + q\mu(p,0) = 0$$

$$-\Delta\sigma^{1/2} + q\sigma^{1/2} = 0$$

Since both $\sigma^{1/2}$ and $\mu \to 1$ at ∞ they are identical.

Main Problem: Given Λ_σ find μ?

1. First find Ψ and hence μ on ∂B by solving

$$[I + S(\Lambda_\sigma - \Lambda_1)]\Psi = \exp(i\zeta \cdot p) \text{ on } \partial B.$$

Here S denotes the operator

$$(Sw)(p) = \int_{\partial B} G(p-t)w(t)ds(t)$$

where G(p) is the Faddeev Greens function

$$-\Delta G = \delta, \quad G \approx \exp(i\zeta \cdot p) \text{ as } |p| \to \infty.$$

2. Compute the "unphysical" scattering transform

$$\mathrm{t}(\mathrm{k}) = \int_{\partial B} \exp(\mathrm{i}\bar{\zeta}\cdot \mathrm{p})\,(\Lambda_{\sigma} - \Lambda_{1})\,\Psi(p)\,ds(p)$$

3. Solve the $\bar{\partial}$ equation for $\mu(\mathrm{p},\zeta)$;

$$\partial\mu / \partial\bar{k} = \frac{1}{4\pi \bar{k}}\, t(k)\exp(i(\zeta + \bar{\zeta})\cdot p)\,\overline{\mu}(p,k)$$

4. Take $\lim_{\mathrm{k}\to 0} \mu(\mathrm{p},\zeta) = \sigma^{1/2}(p)$

5. Display σ.

Does it Work?

Numerical Simulation

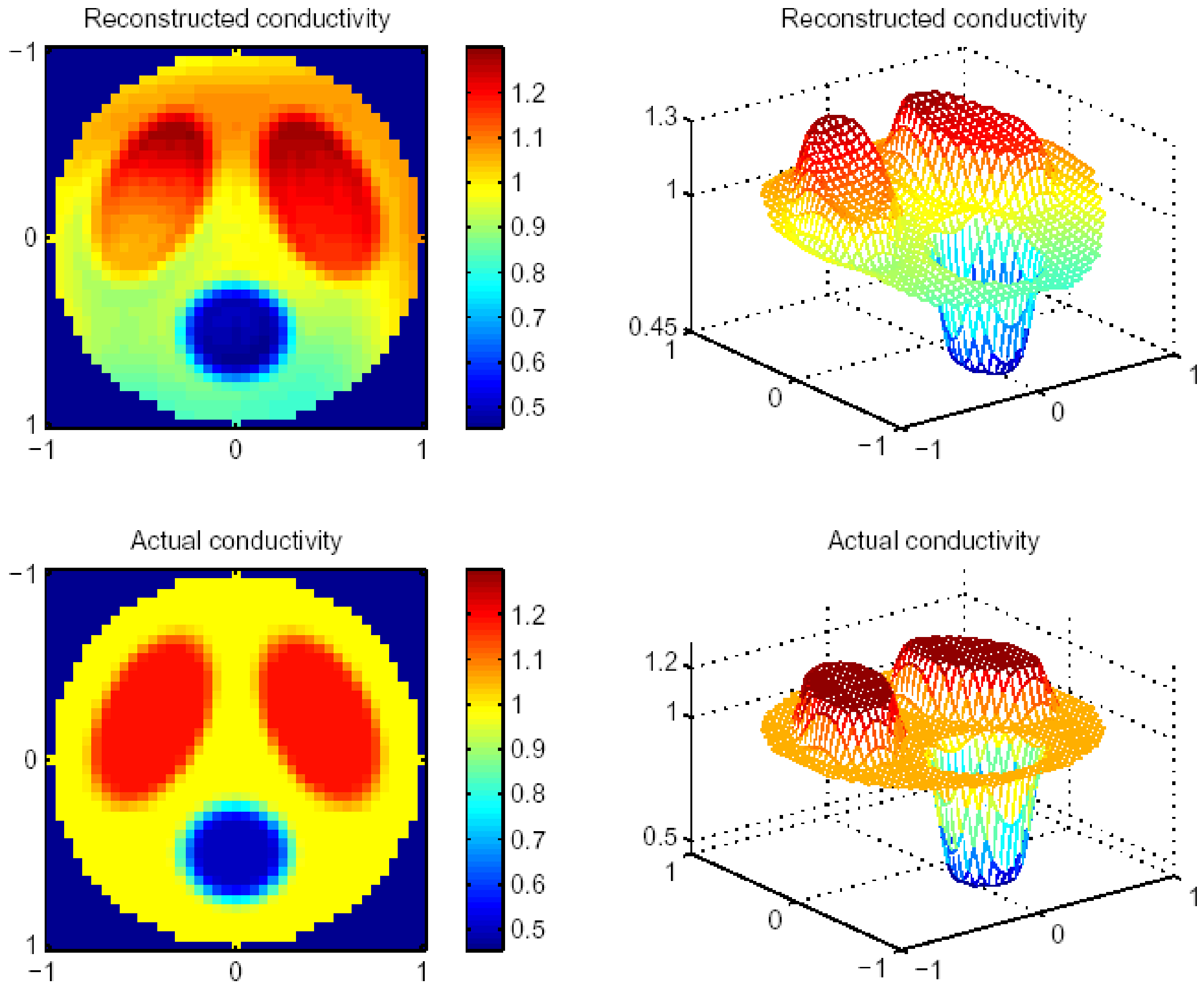
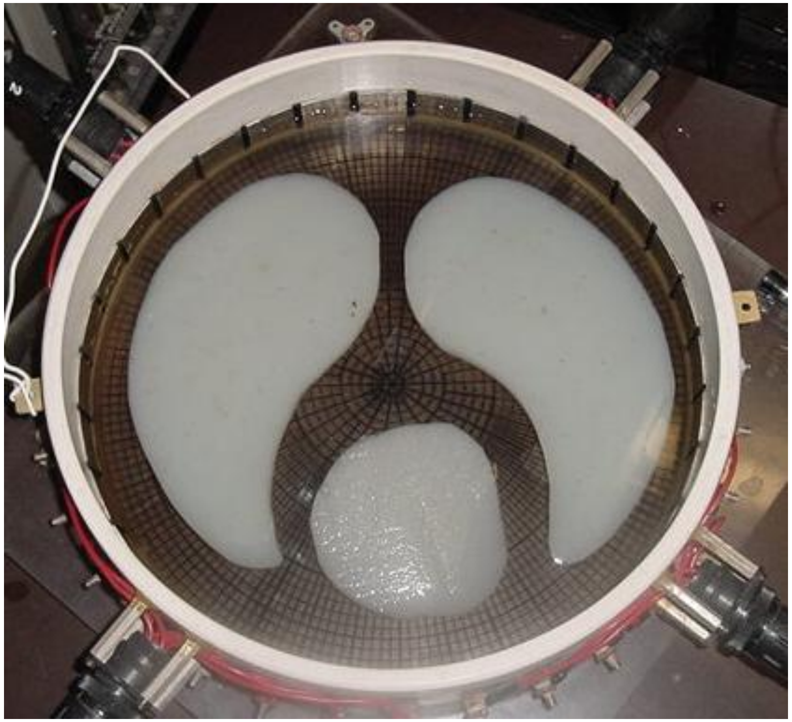
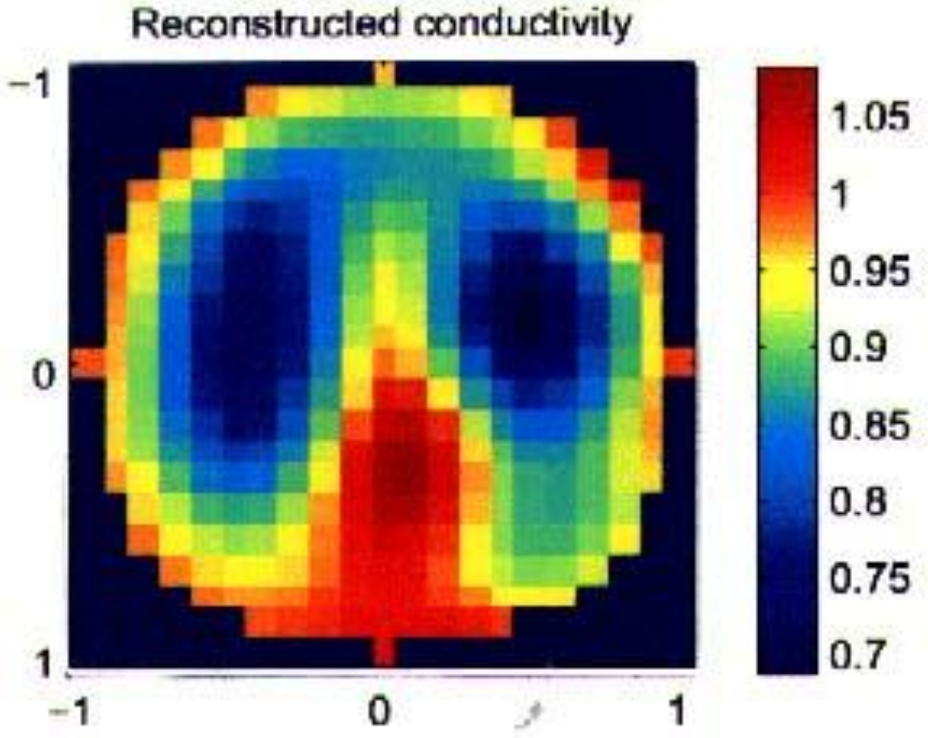


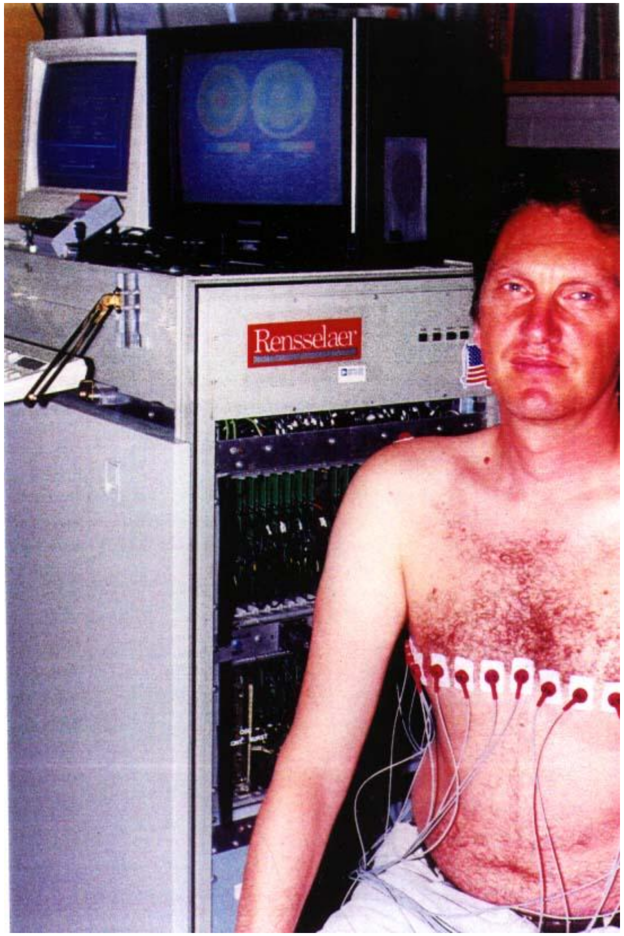
FIG. 5.2. *Plots of the actual and reconstructed conductivities for the virtual phantom chest.*

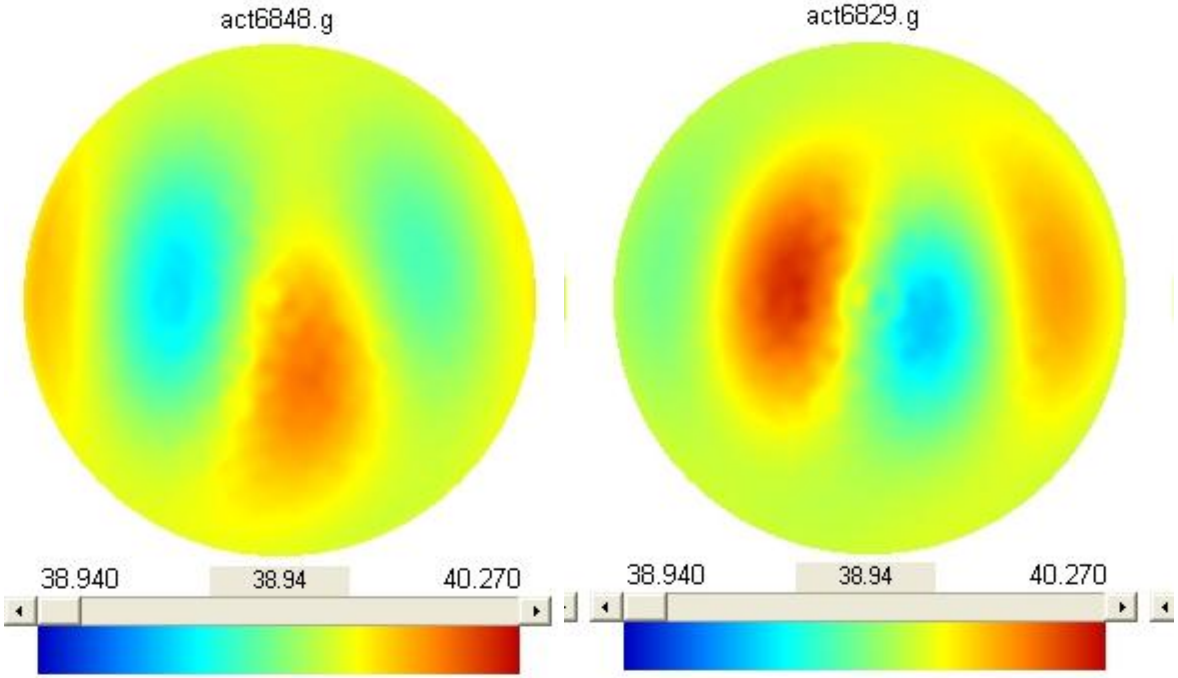
First D-Bar Reconstruction Results from Experimental Data





First D-Bar Cardiac Images

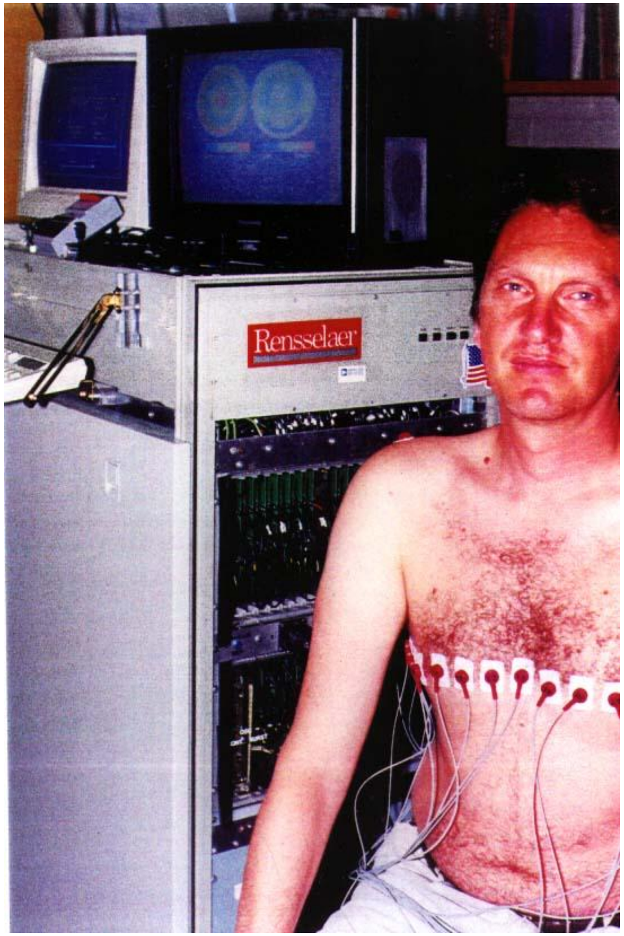


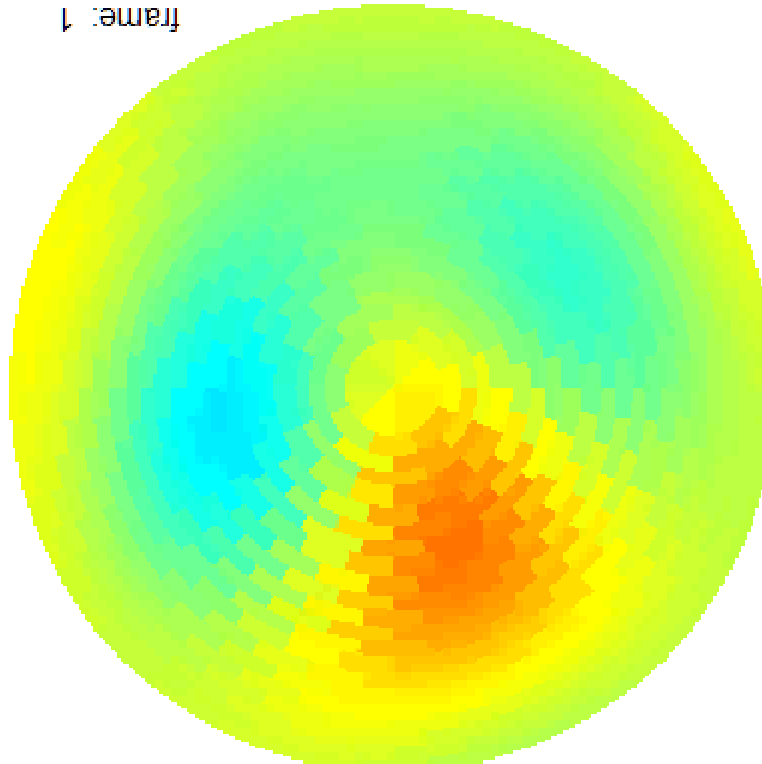


Changes in conductivity as heart expands (diastole) and contracts (systole) from one fixed moment in cardiac cycle.

First blood fills enlarging heart (red) while leaving lungs (blue) . Then blood leaves contracting heart (blue) to fill lungs (red).

Reconstruction by D-bar. Data by ACT3.





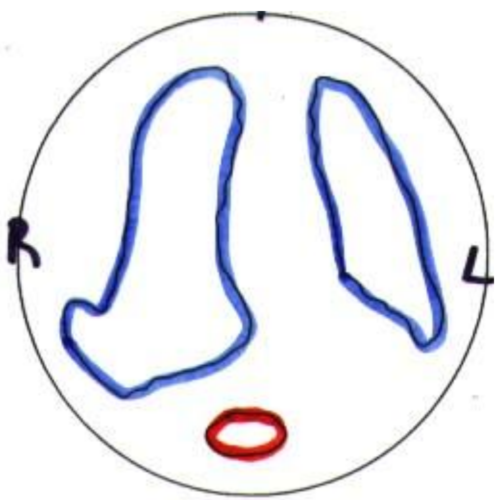
Click on the image at right to see a movie of changes in the conductivity inside a chest during the cardiac cycle. Difference's shown in the movie are all from one moment in the cycle. The movie starts with the heart filling and the lungs emptying.

Reconstruction by D-Bar. Data from ACT3.

Regions of interest: lungs and heart

Posterior

Right



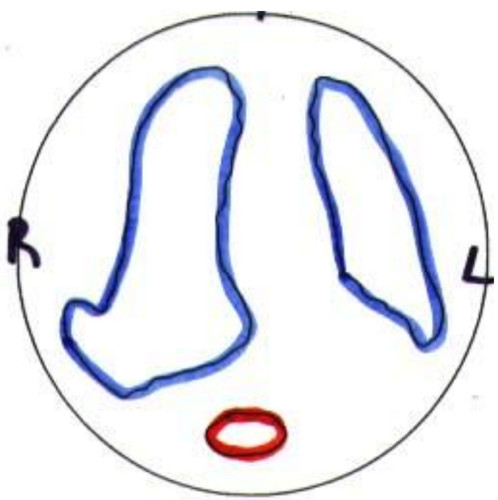
Left

Anterior

Regions of interest: lungs and heart

Posterior

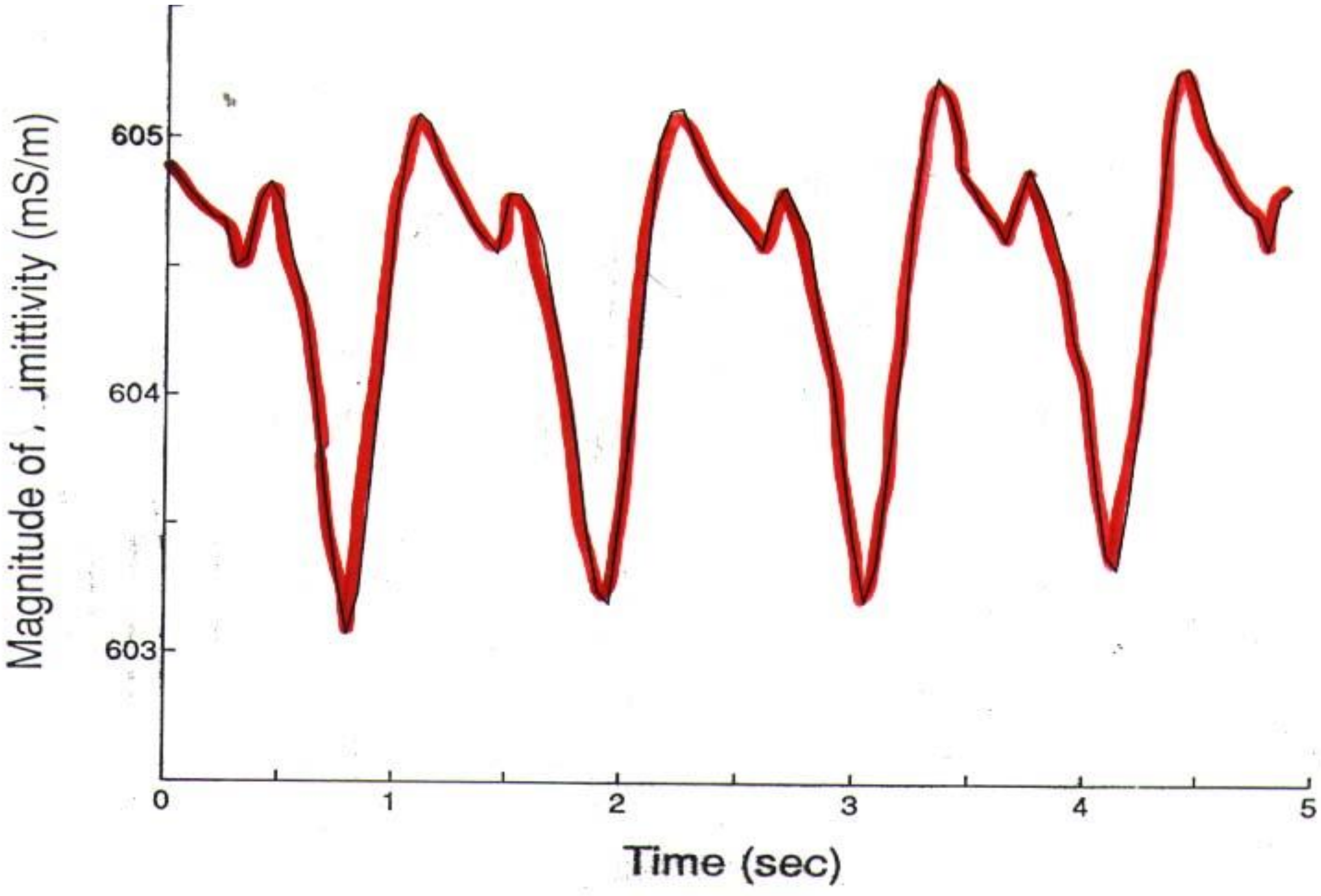
Right



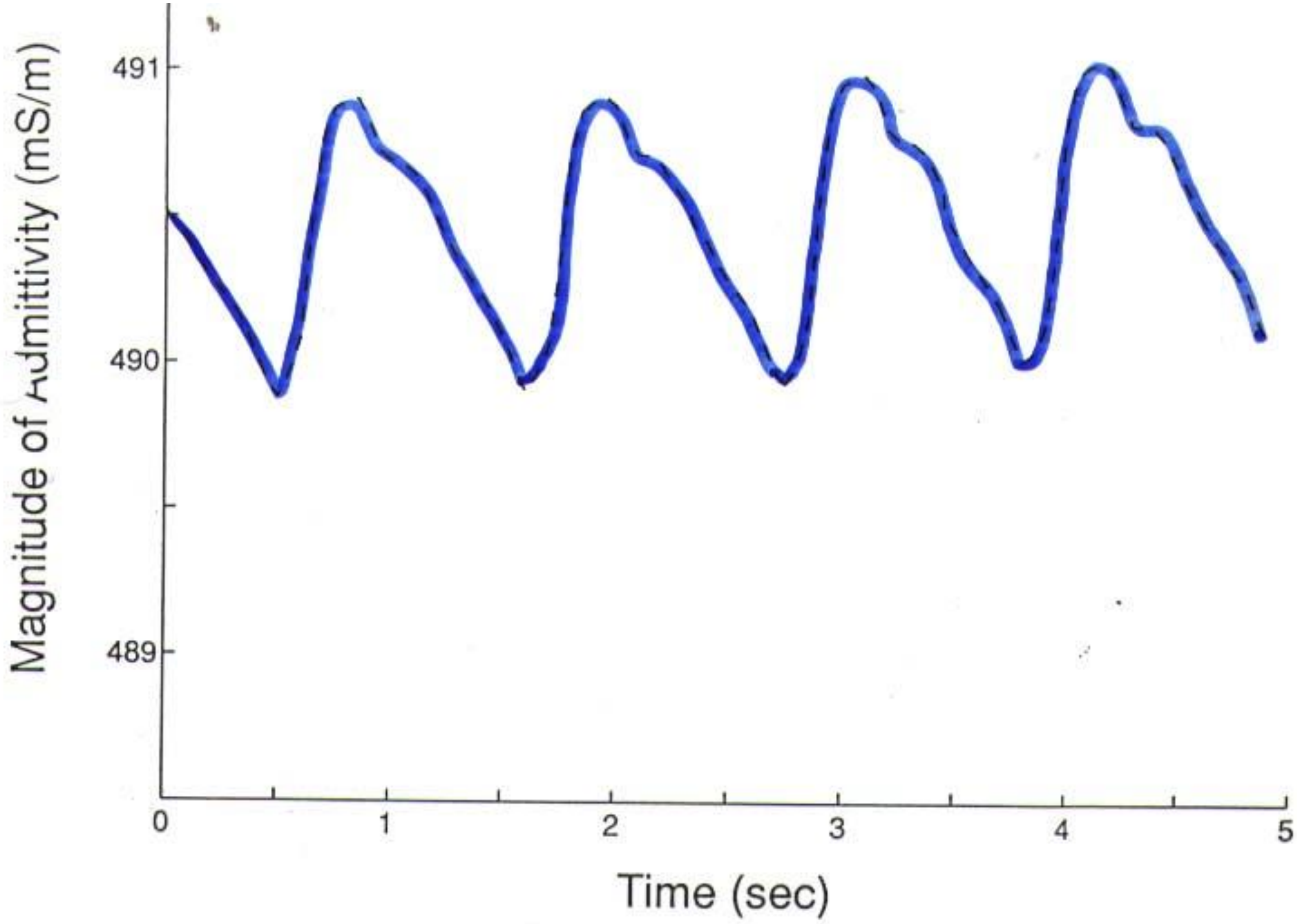
Left

Anterior

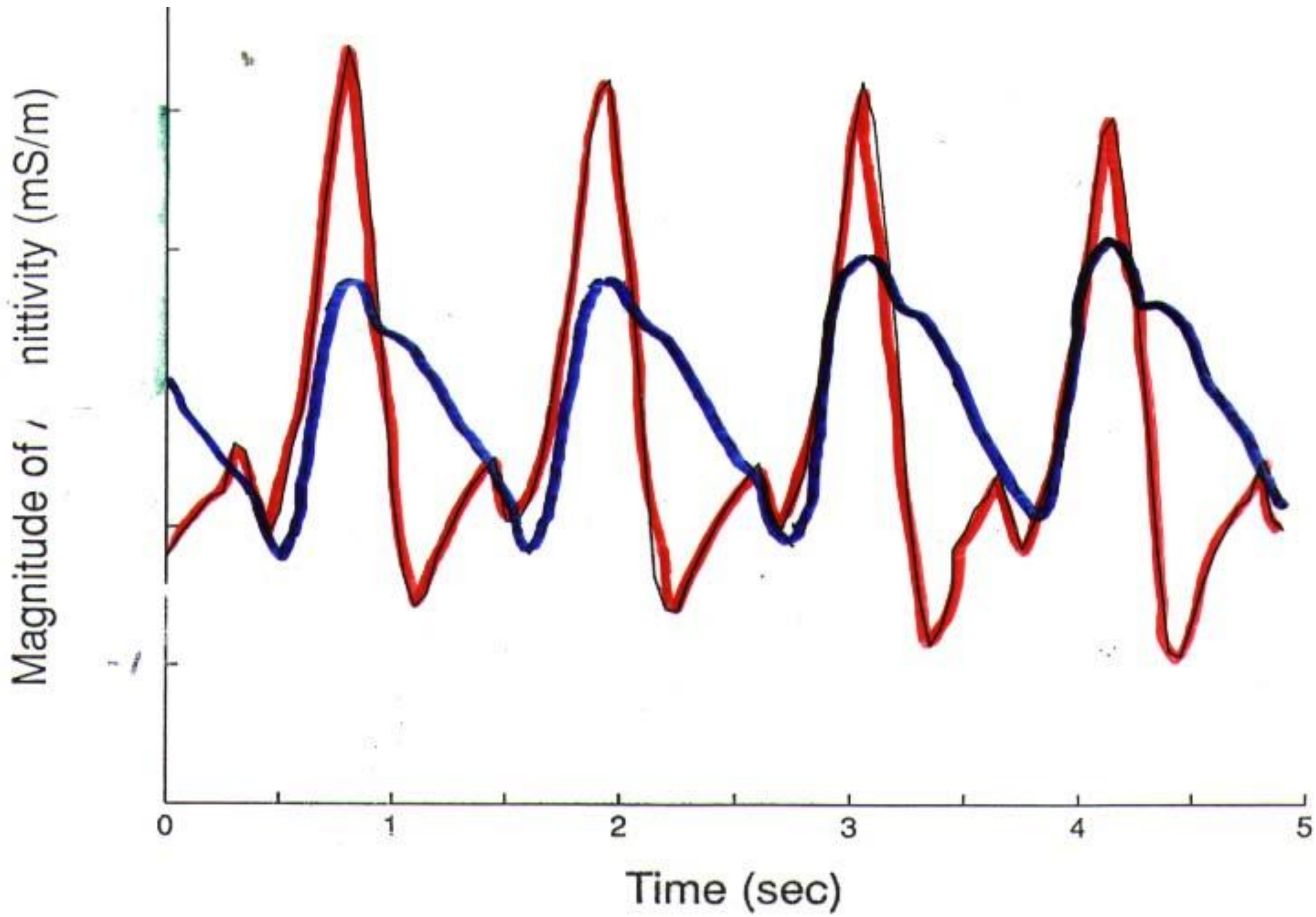
Admittivity of the heart region.



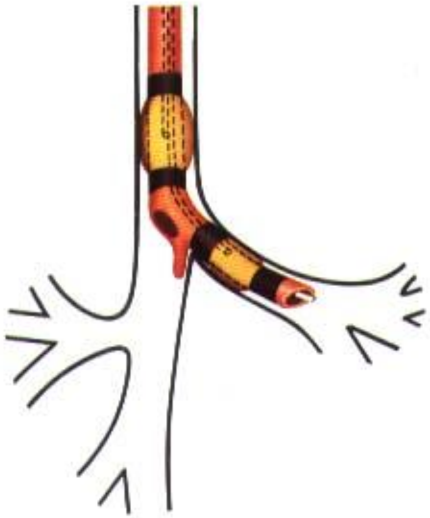
Admittivity of the lung region.



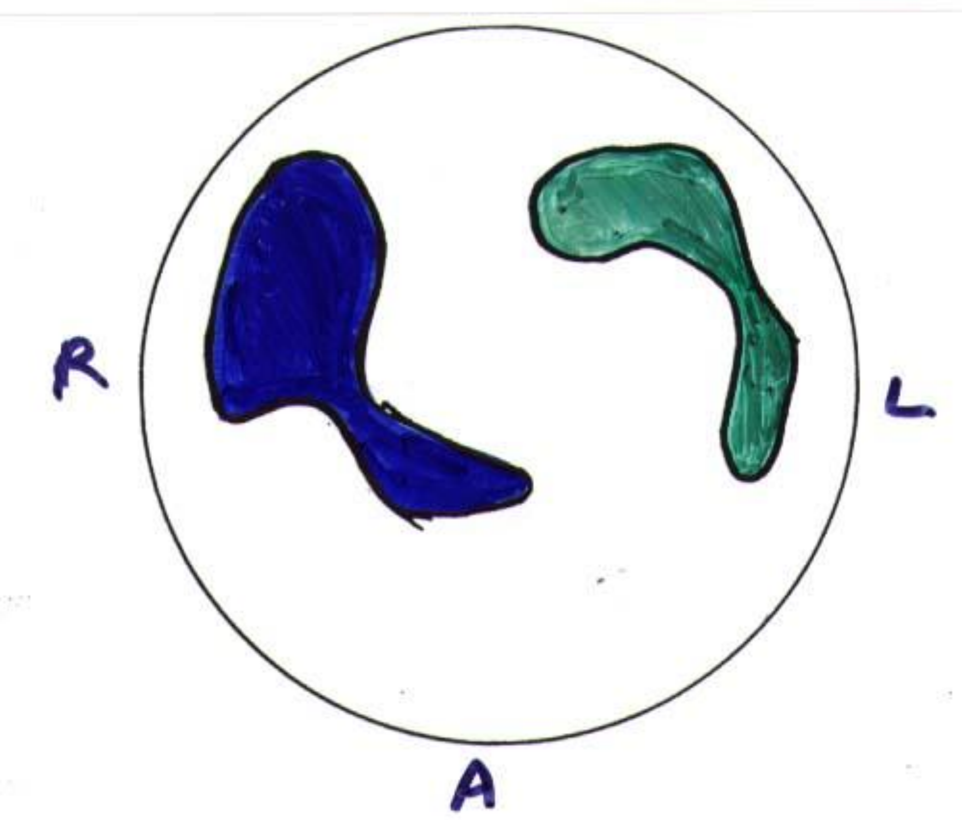
Admittivity of the lung region (blue) and heart region (red, inverted scale).



Tracheal Divider



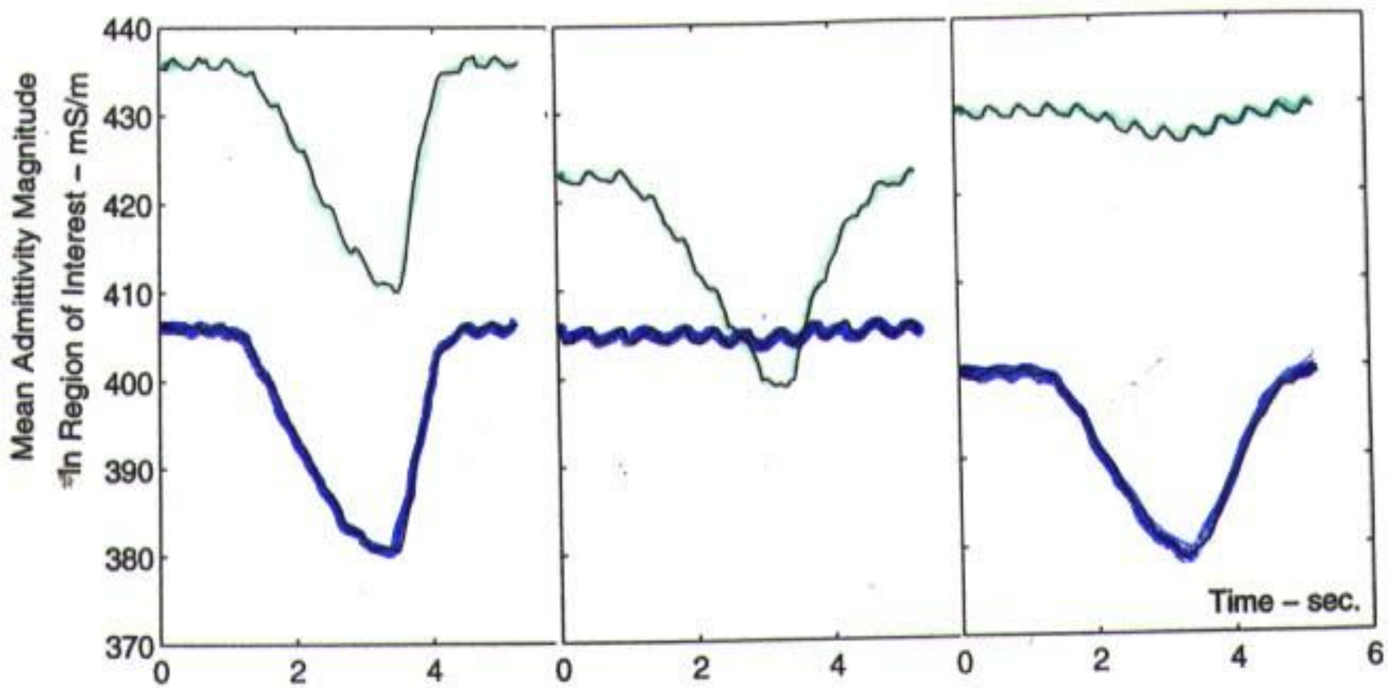
Regions of interest over the right and left lungs.



Admittivity of the left and right lungs during ventilation of both lungs, then left lung only, then right lung only.

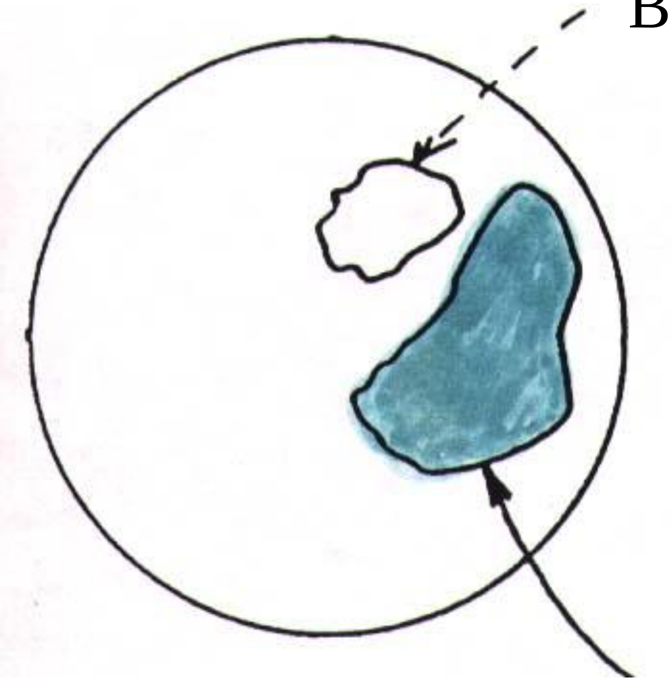
Left

Right

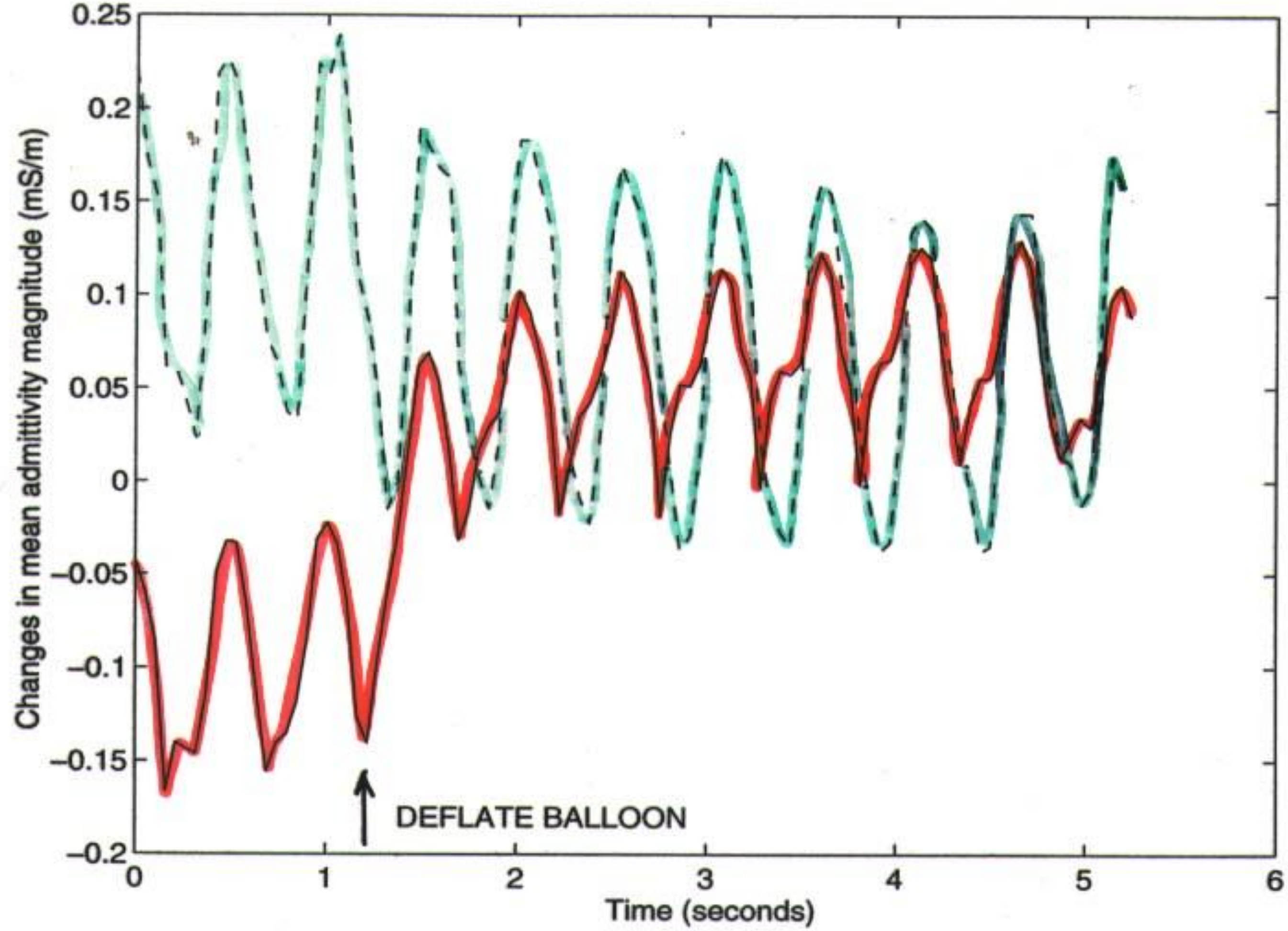


Ventilation of: Both Left Right

Regions of interest over the lung.



Changes in admittivity with deflation of a balloon in a branch of the pulmonary artery.



How to image σ better?

The Holy Grail:

How to image J^H in real time at a microscopic scale?

Hybrid methods?

CDI, MREIT,PAT,TAT, AMEIT…

New ideas are needed!

Thank you!

Especially

S. McD., P.S., A.V., L.W., M.Z,

and

G.U.

Lunch time!

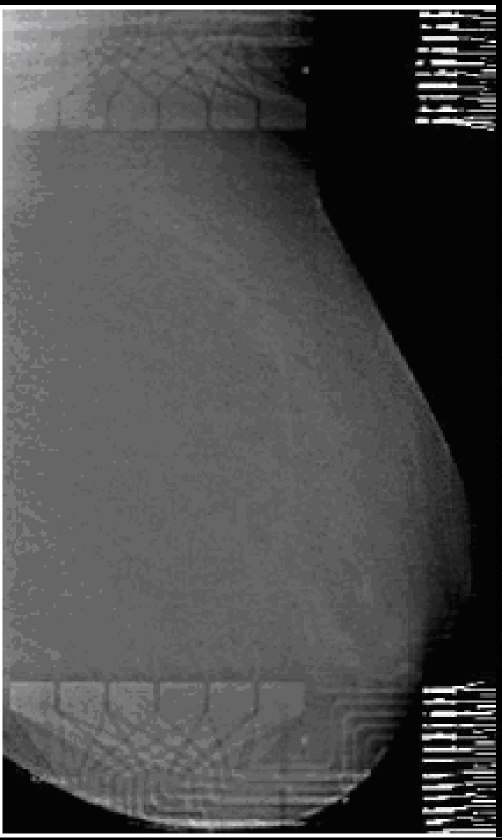
Problems:

- How to make D-bar method work better with experimental data?
- How to make it work in 3-D?
- How to make D-bar work with Optical,Acoustic, and Microwave Data?

Can EIT Improve Sensitivity and Specificity in screening for Breast Cancer ?

Breast Cancer Problem

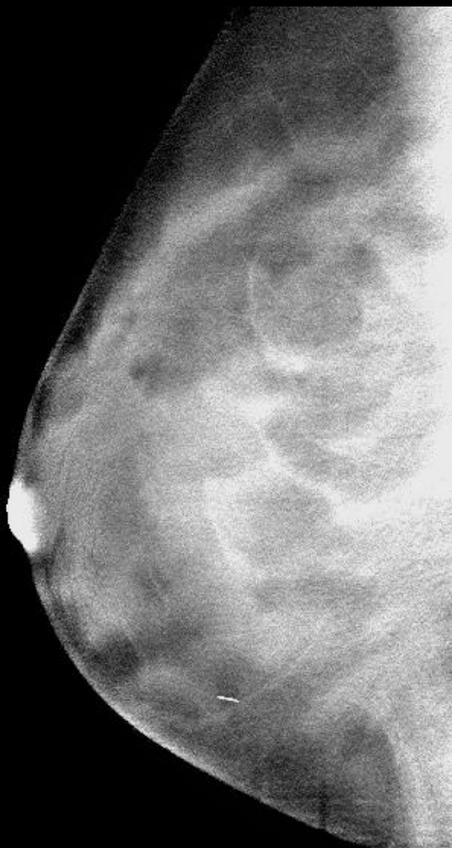
Which ones have cancer ?



HS14R

HS21R

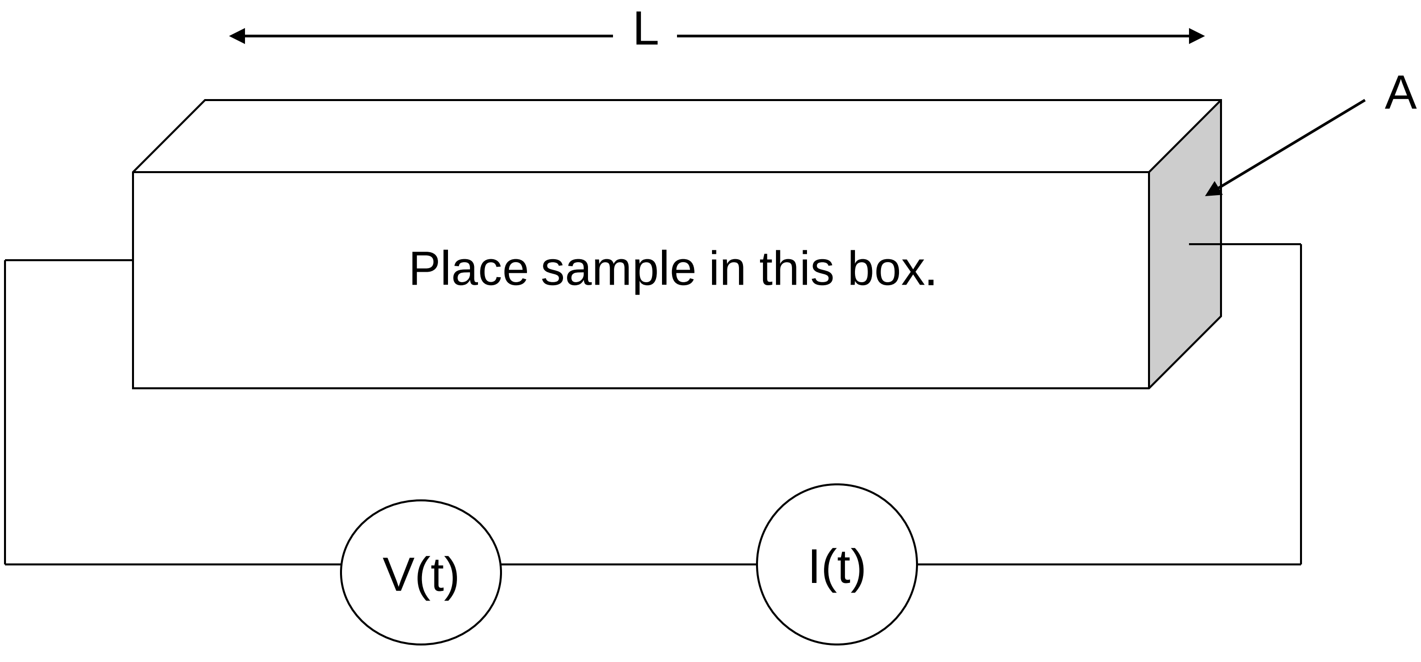
HS25L



HS10L

Observation of Jossinet;
Electrical Impedance Spectra can distinguish different tissues.

How to measure Impedance Spectra.



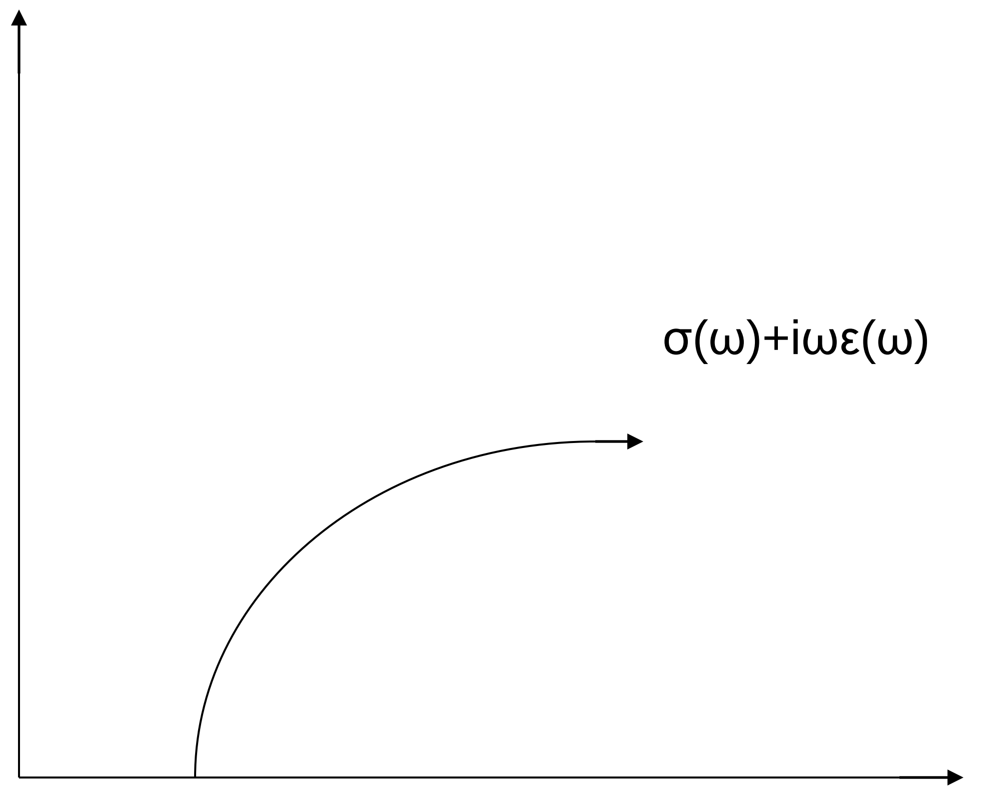
Apply voltage, $V(t) = V\cos(\omega t) = \mathrm{Re}\,[\, V \exp(i\omega t)\,]$.

Measure current, $I(t) = V\,(\, a\cos(\omega t) - b\sin(\omega t)\,) = \mathrm{Re}\,[V(a+ib)\exp(i\omega t)\,]$.

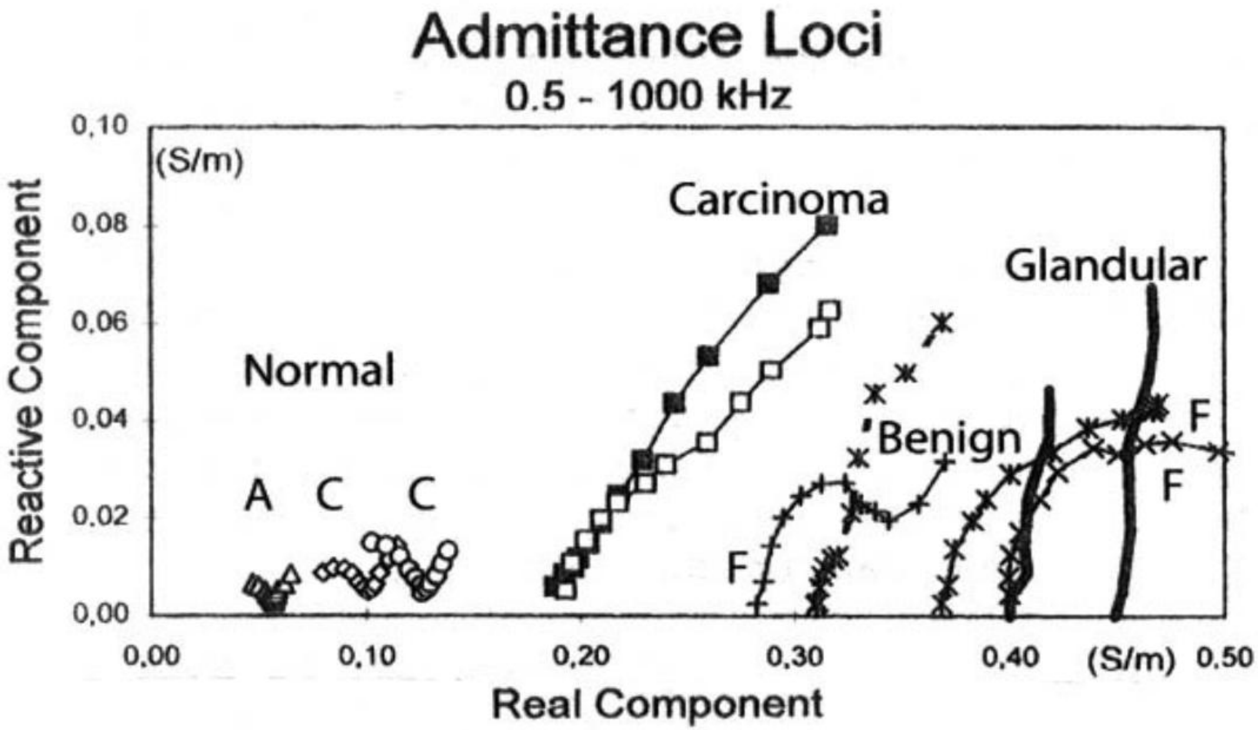
$\sigma + i\omega\varepsilon \equiv (a+ib)(L/A)$

How we plot electrical impedance spectra in each voxel.

Electrical Impedance Spectra, EIS Plot, of admittivity, $\sigma(\omega)+i\omega\varepsilon(\omega)$, for 5kHz <ω<1MHz.



Admittance Loci: format for summaries of EIS data



Results of in-vitro studies of excised breast tissue. *Jossinet & Schmitt 1999*

Electrical Impedance Tomography with Tomosynthesis for Breast Cancer Detection

Jonathan Newell

With:

David Isaacson
Tzu-Jen Kao
Richard Moore*
Gary J. Saulnier
Greg Boverman
Daniel Kopans*

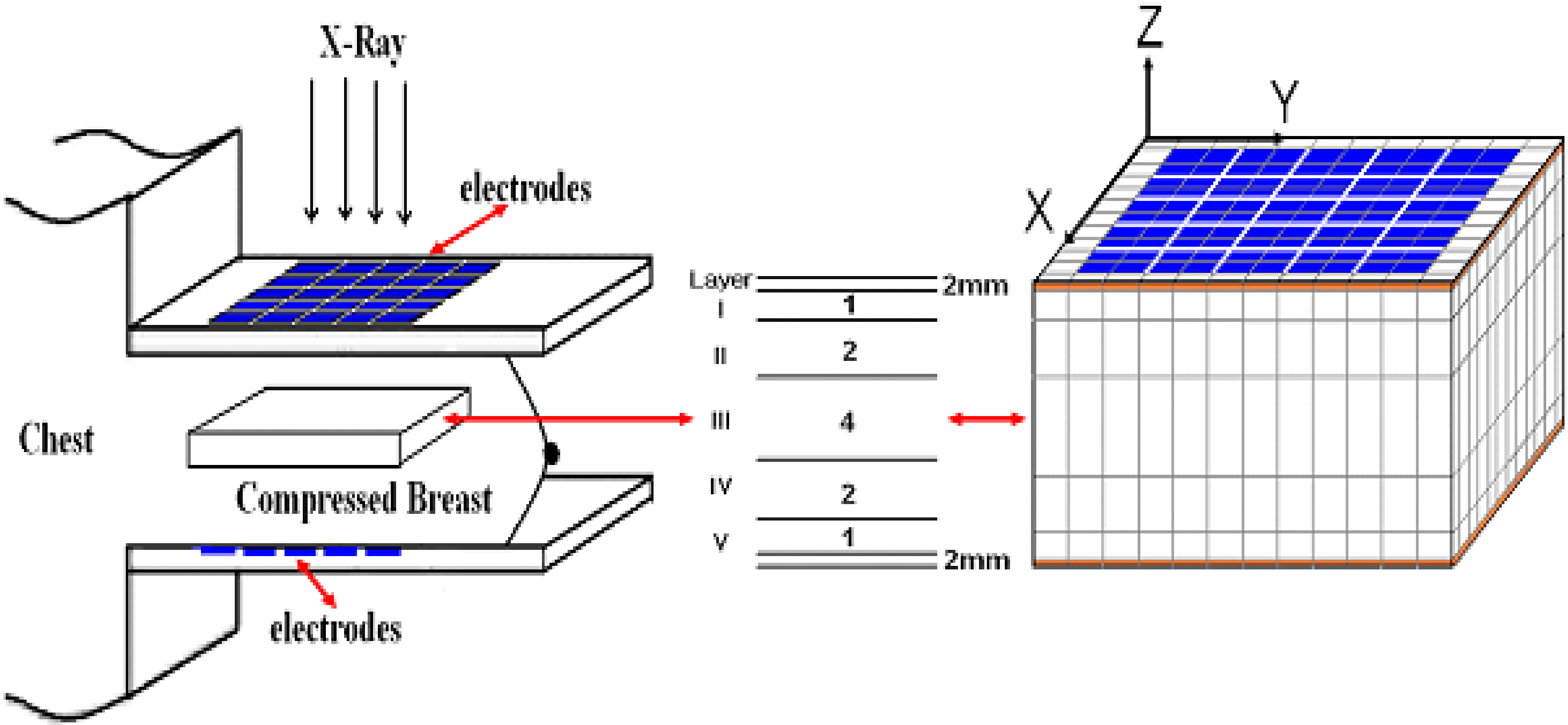
And:

Rujuta Kulkarni
Dave Ardrey
Chandana Tamma
Neha Pol

Rensselaer Polytechnic Institute

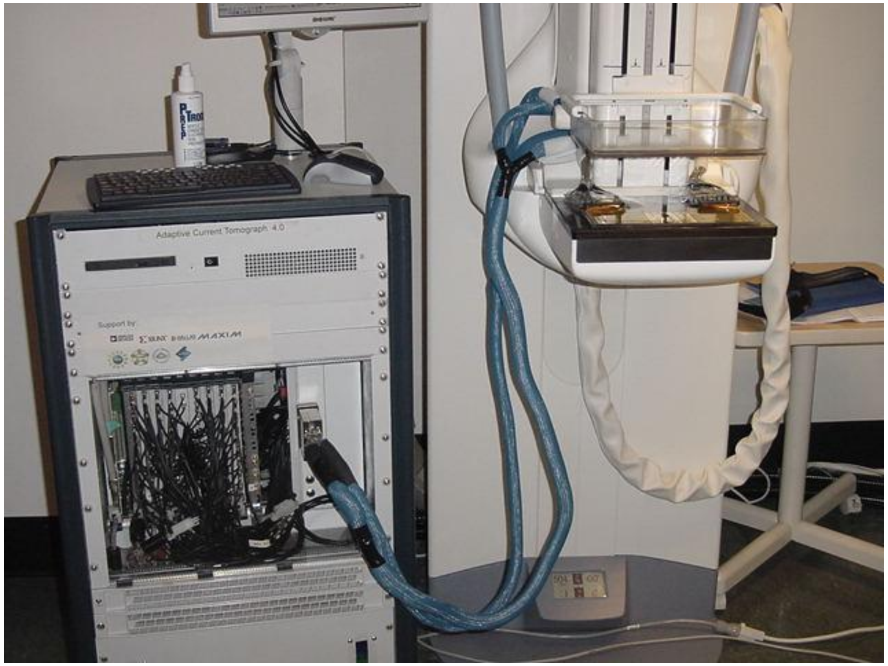
*Massachusetts General Hospital

EIT electrodes added to mammography machine.

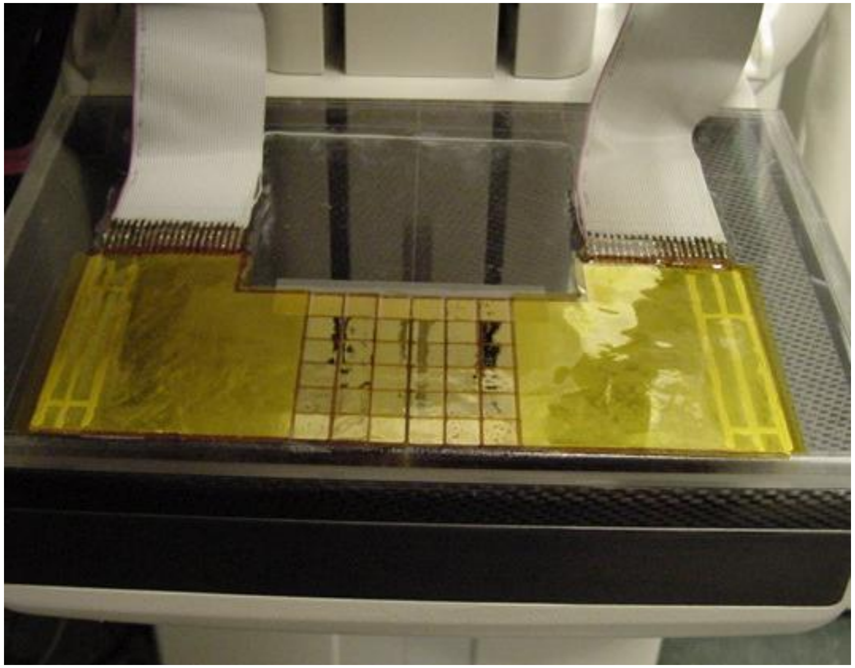


- 1 : 2 : 4 : 2 : 1 is the ratio of the mesh thicknesses.
- Only the center layer, III, is displayed in the results.

EIT Instrumentation

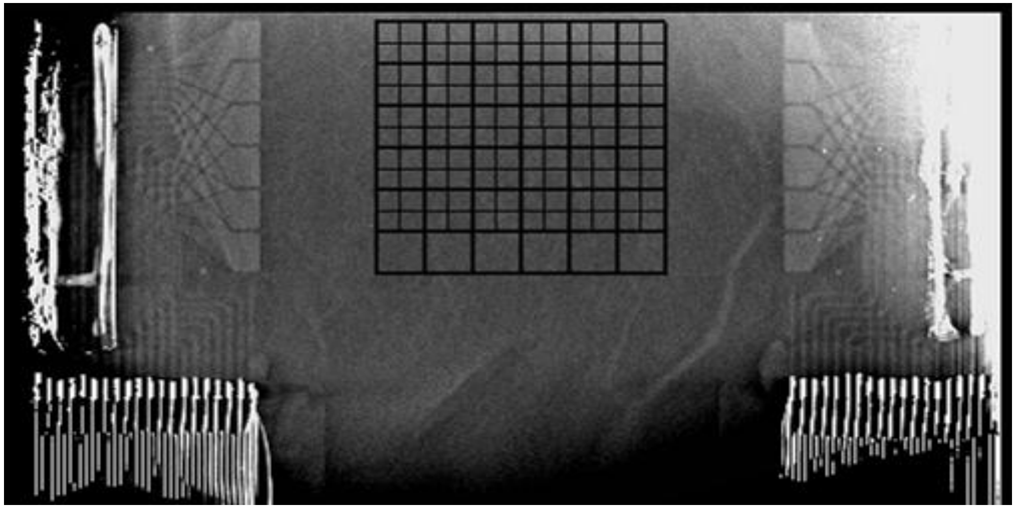


ACT 4 with Tomosynthesis unit



Radiolucent electrode array

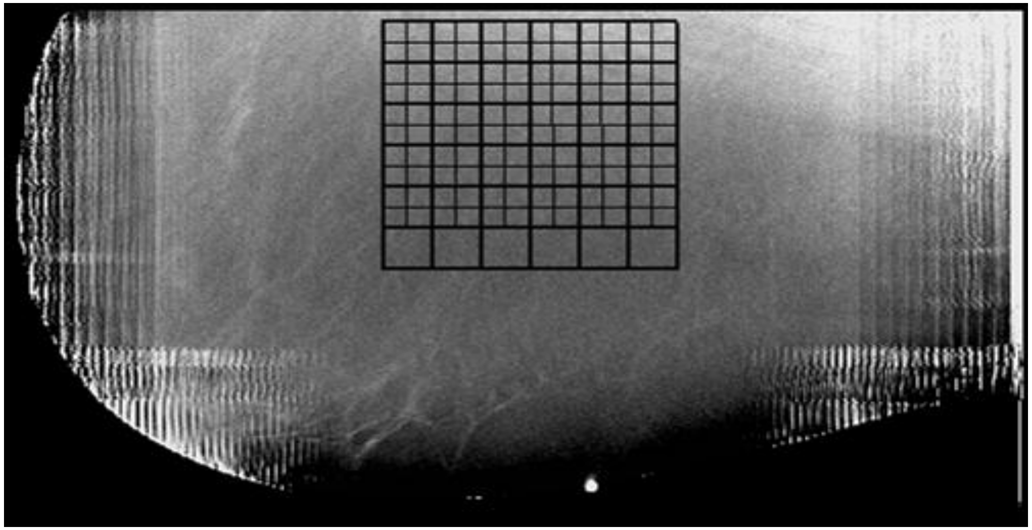
Co-registration of EIT and Tomo Images



To find the electrode position, display the slice containing the electrodes.
Superimpose the mesh grid with correct scale.

Slice 15 of 91

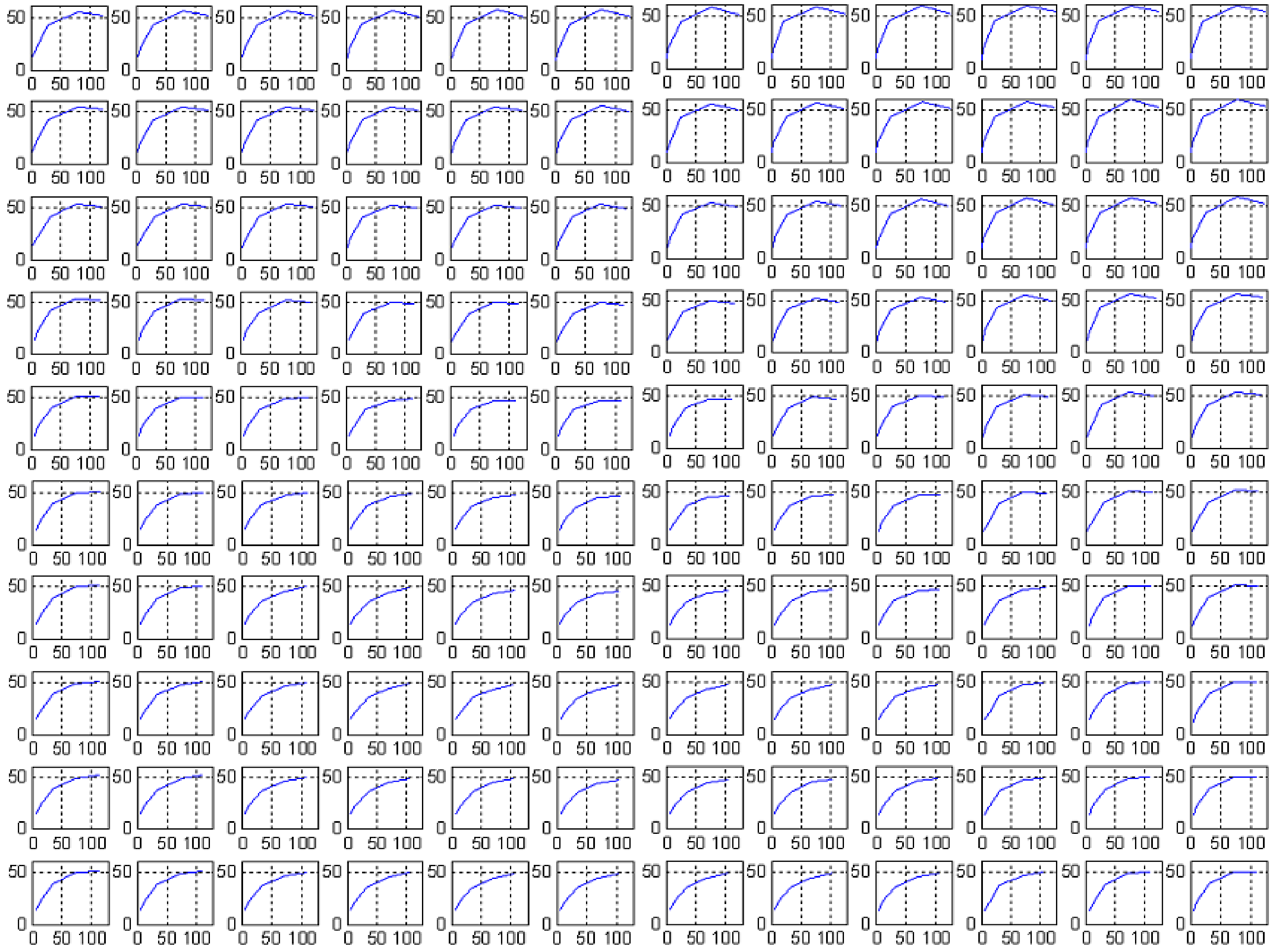
HS_14R
Normal



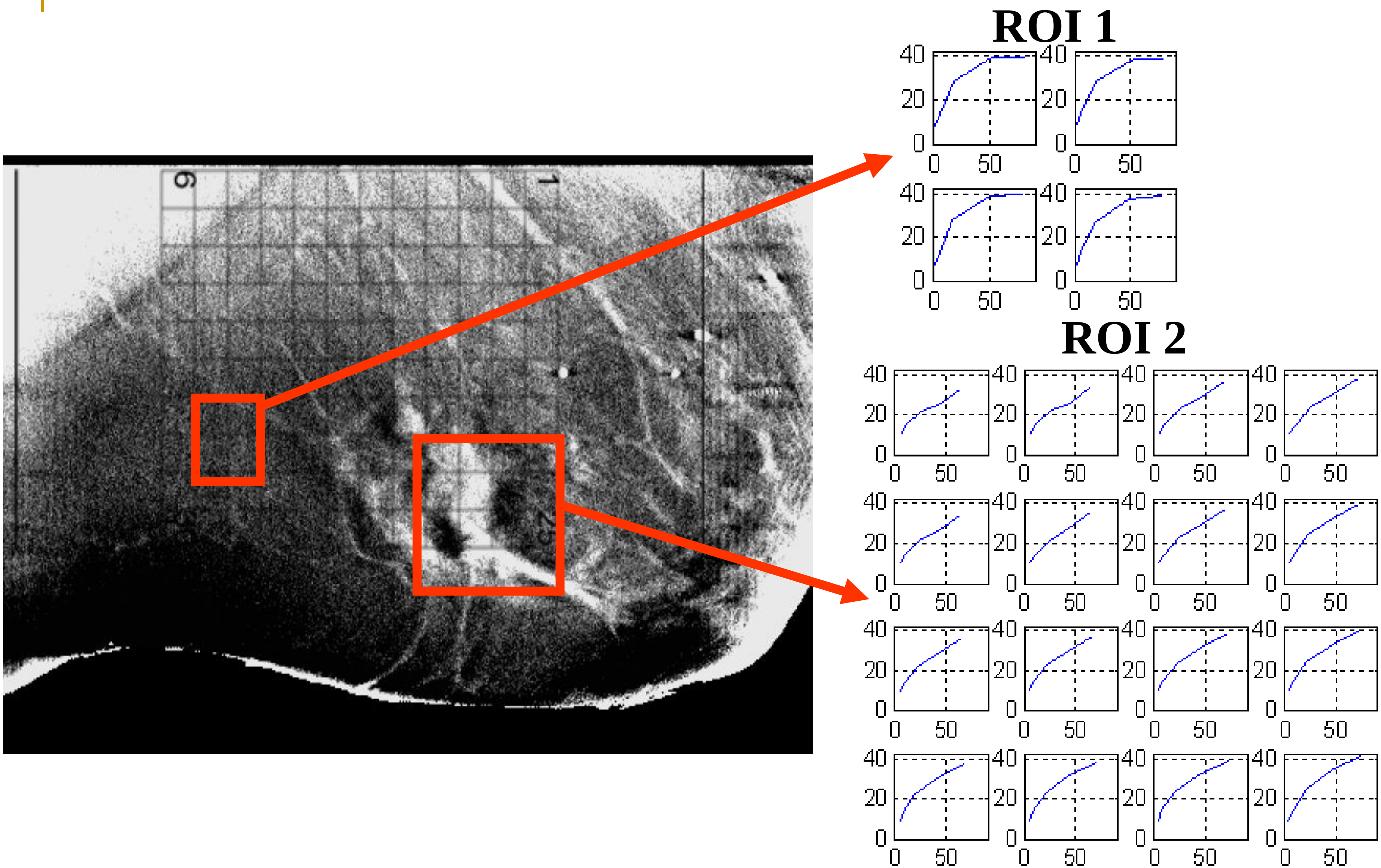
Then select the desired tomosynthesis layer.

Slice 50 of 91

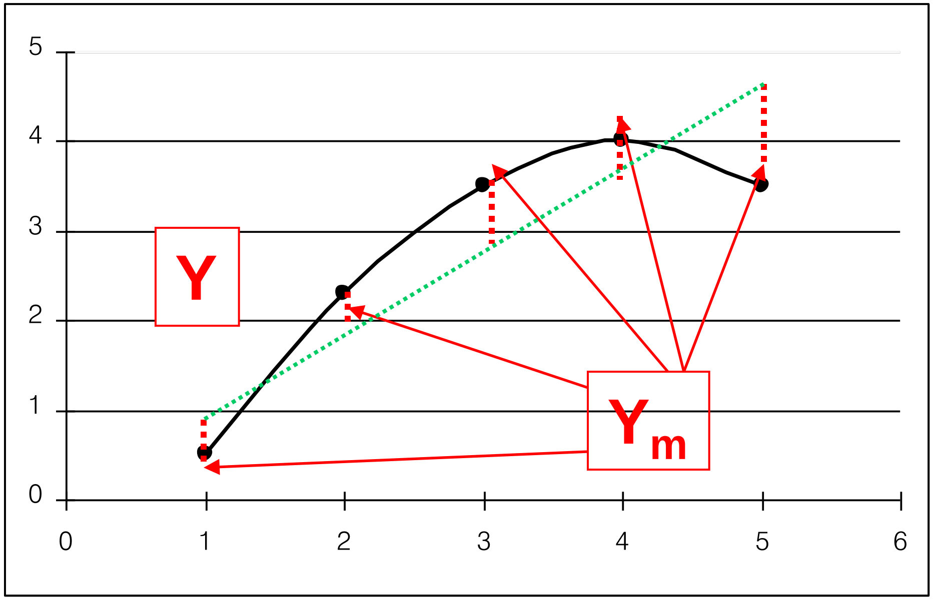
120 EIS plots for a normal breast (HS14_Right)



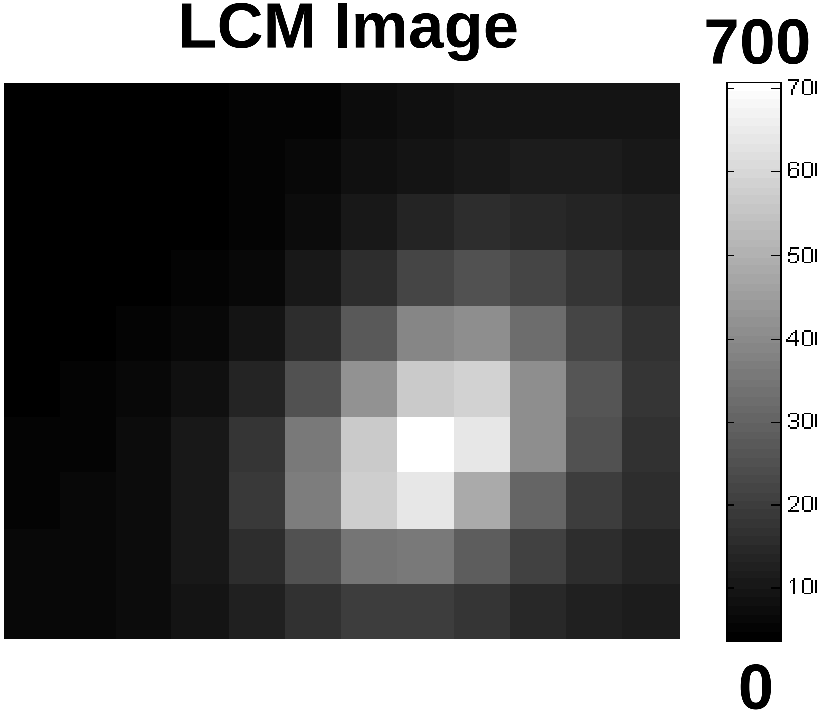
HS25_L: Invasive Ductal Carcinoma



Linear Correlation Measure –LCM



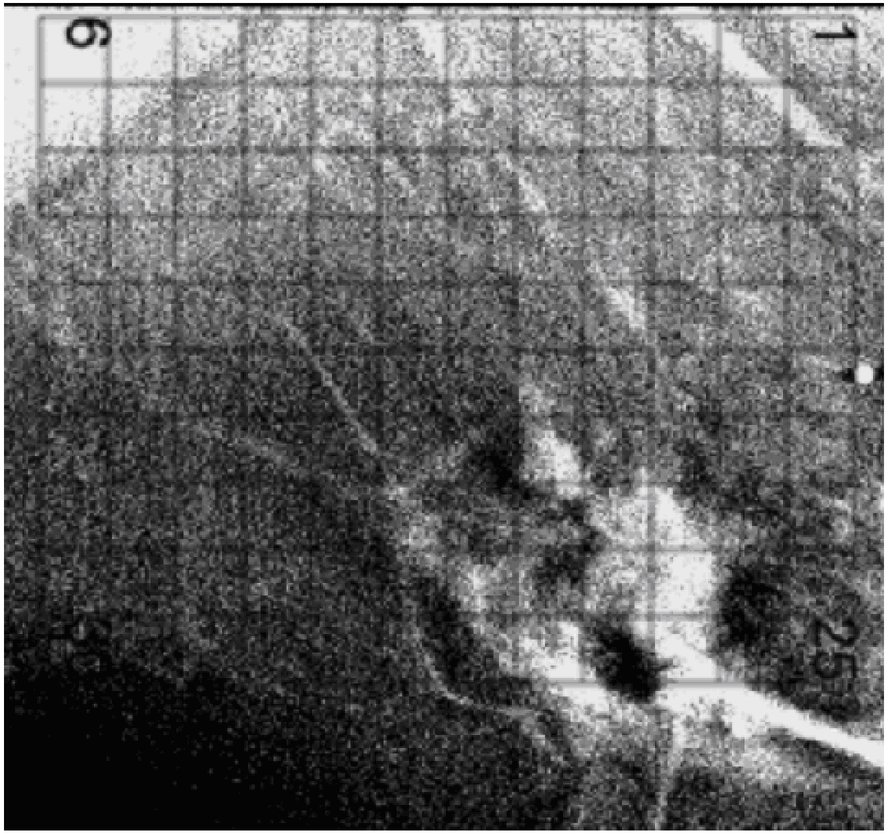
LCM Image

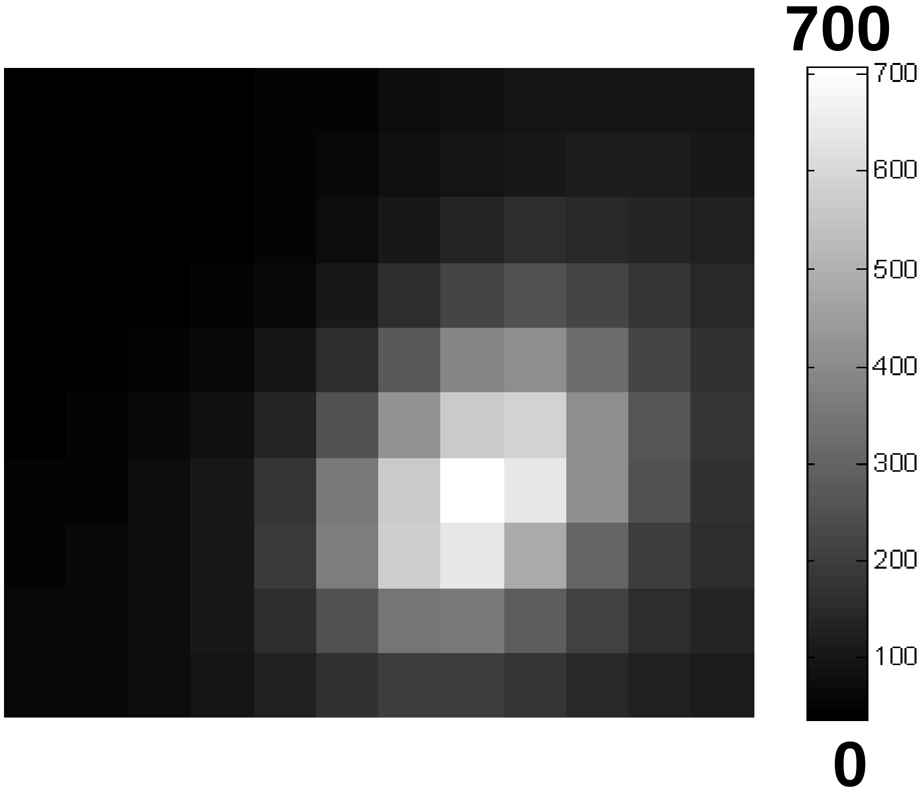


$$LCM = \frac{1}{1 - \frac{\left|\langle Y, Y_m \rangle\right|}{\|Y\| \|Y_m\|}}$$

compute for each voxel

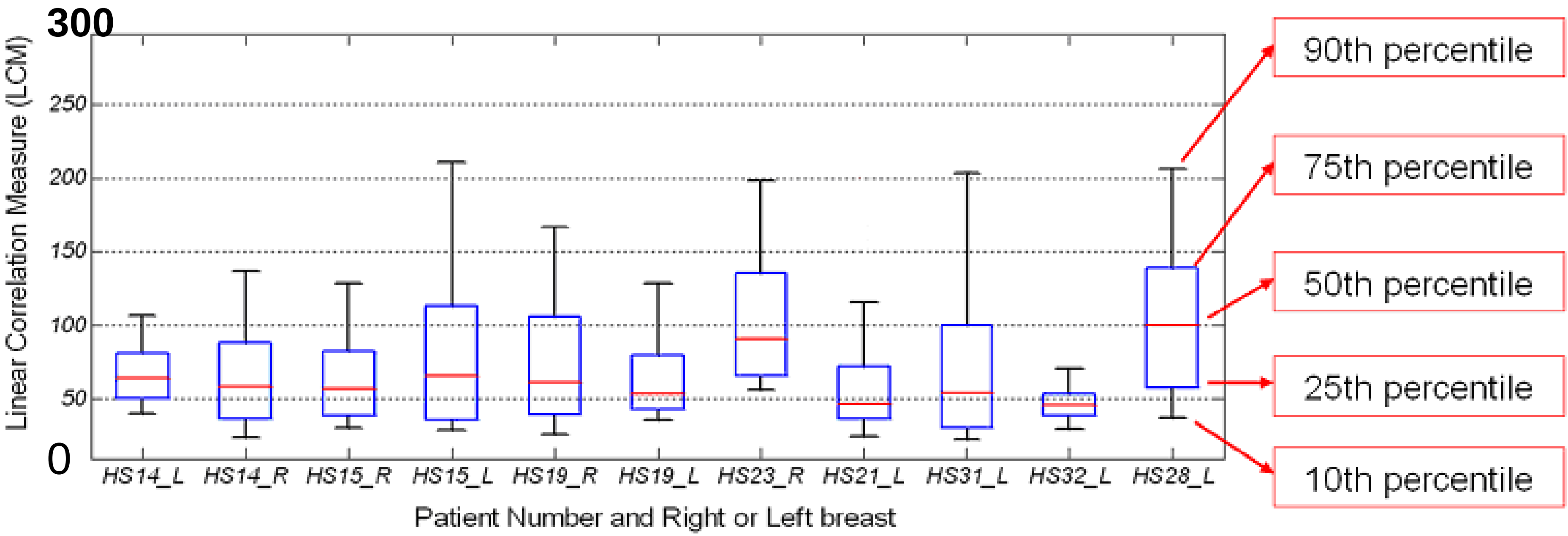
LCM Image of invasive ductal CA (HS25_L)





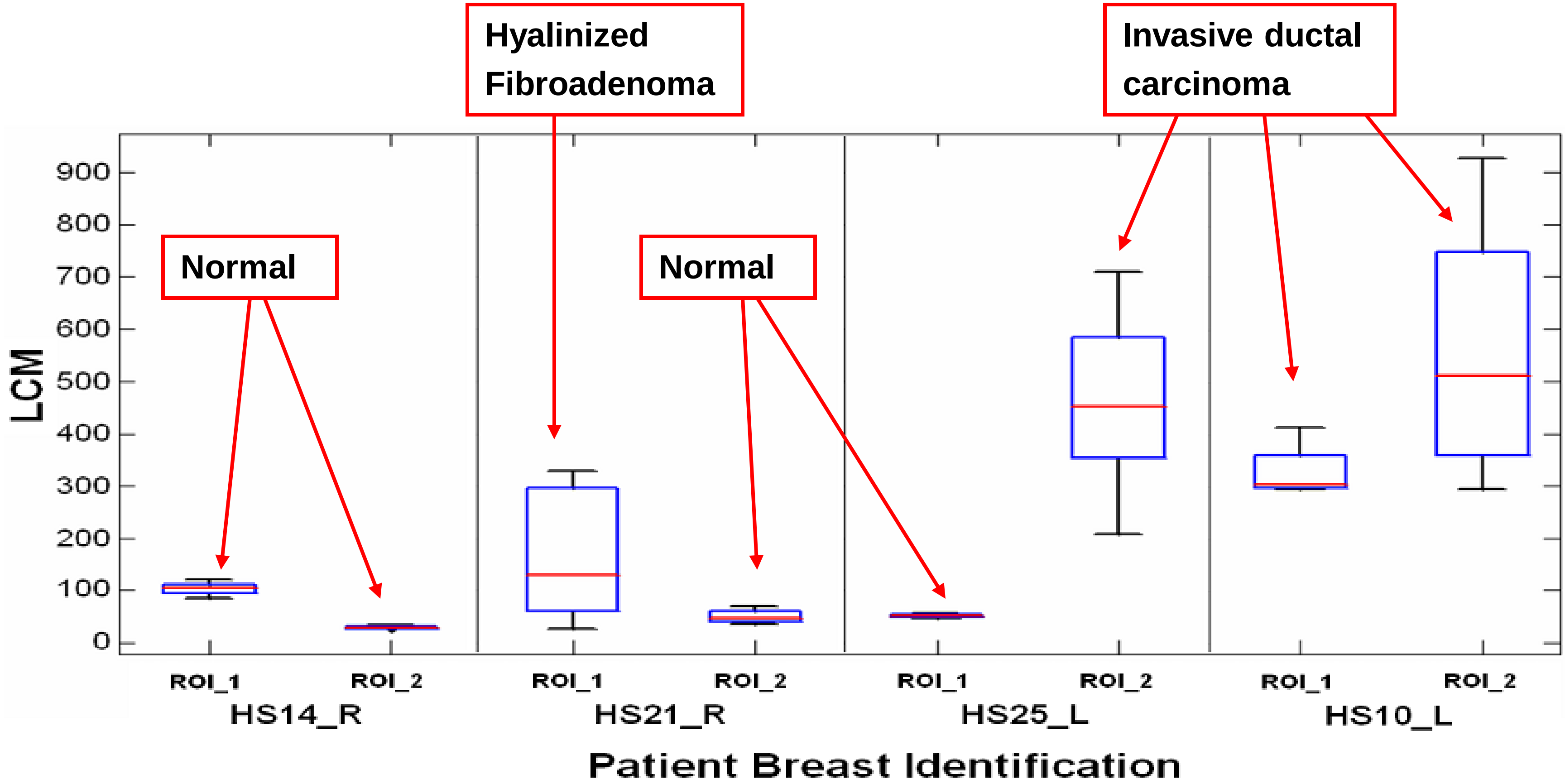
Gray scale image of LCM

LCM for 11 normal breasts



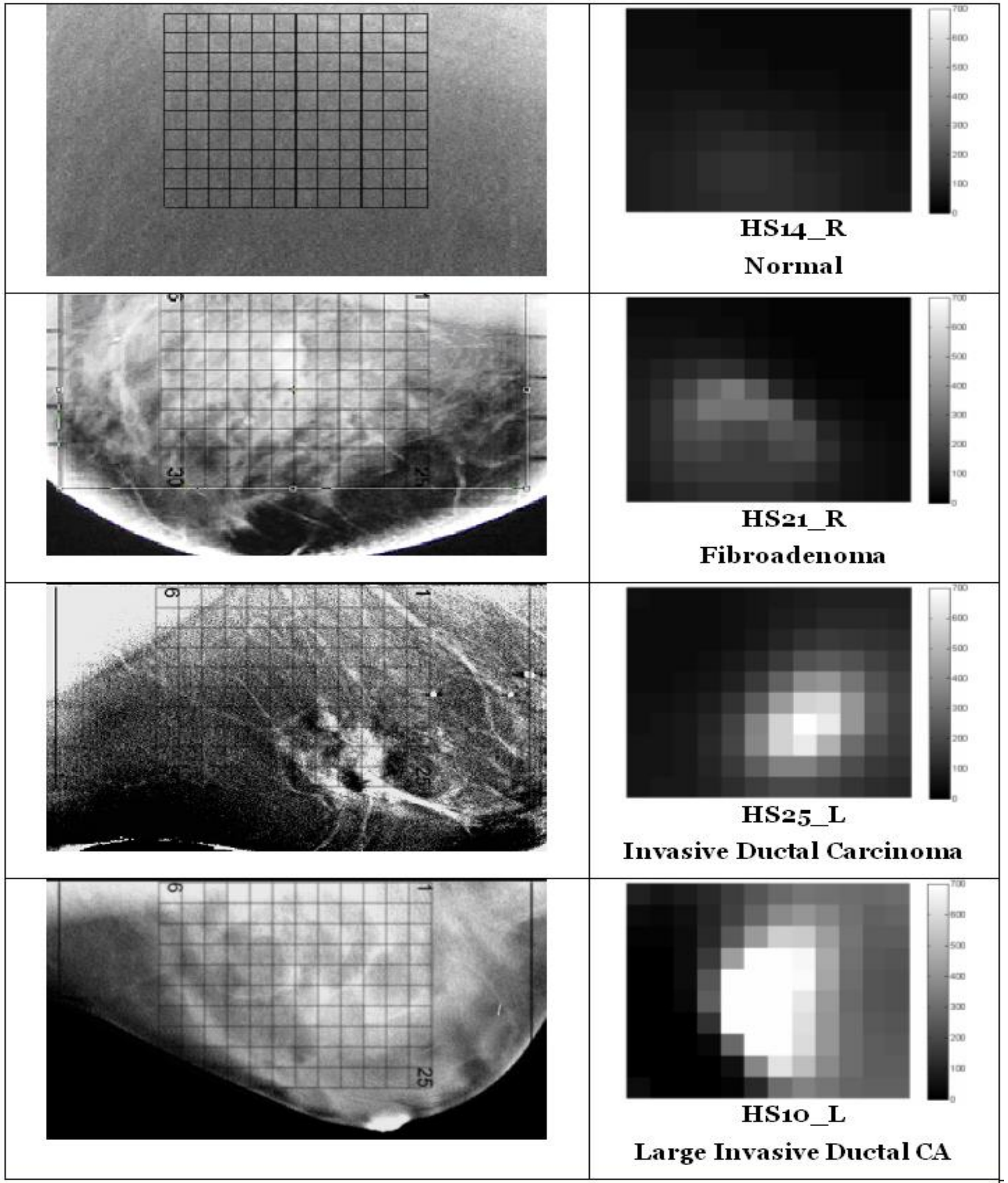
There are 120 EIS plots for layer 3 in each patient. The distribution of the LCM parameter in these plots is shown.

LCM for the regions of interest in 4 patients



The distributions of the LCM for the regions of interest identified. Note the LCM values are much larger for voxels associated with the malignant lesions.

LCM on the same scale



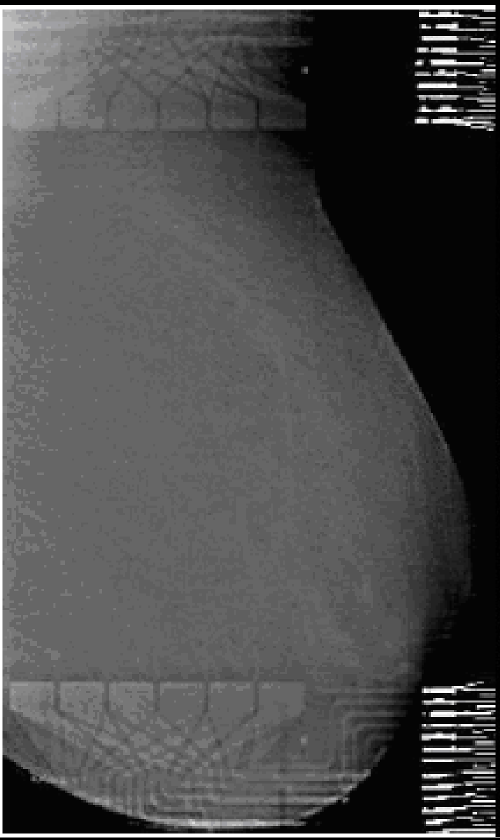
Normal Breast

Fibroadenoma

Invasive Ductal Carcinoma

Invasive Ductal Carcinoma

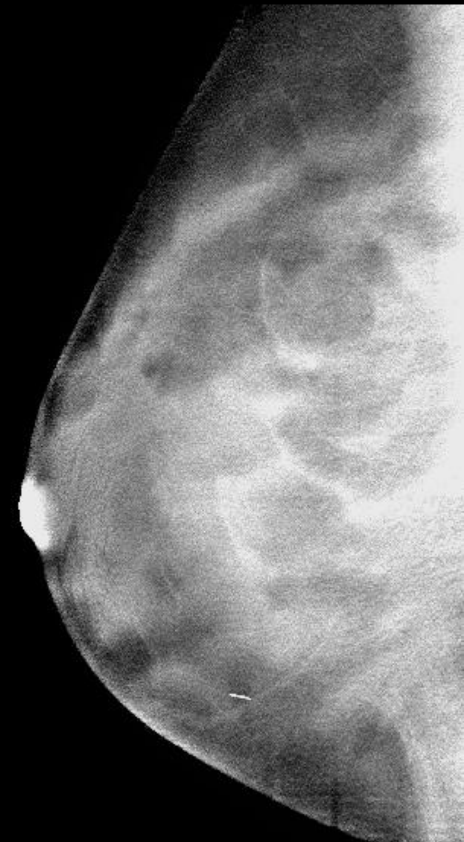
Which ones have cancer ?



HS14R

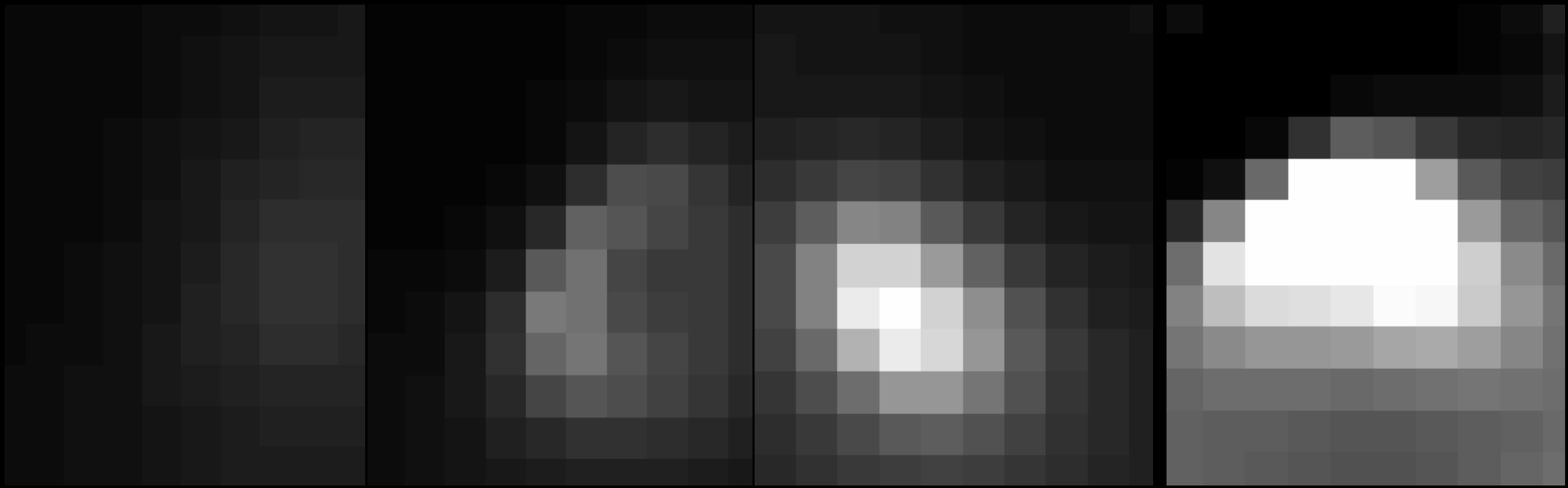
HS21R

HS25L



HS10L

Which ones have cancer ?



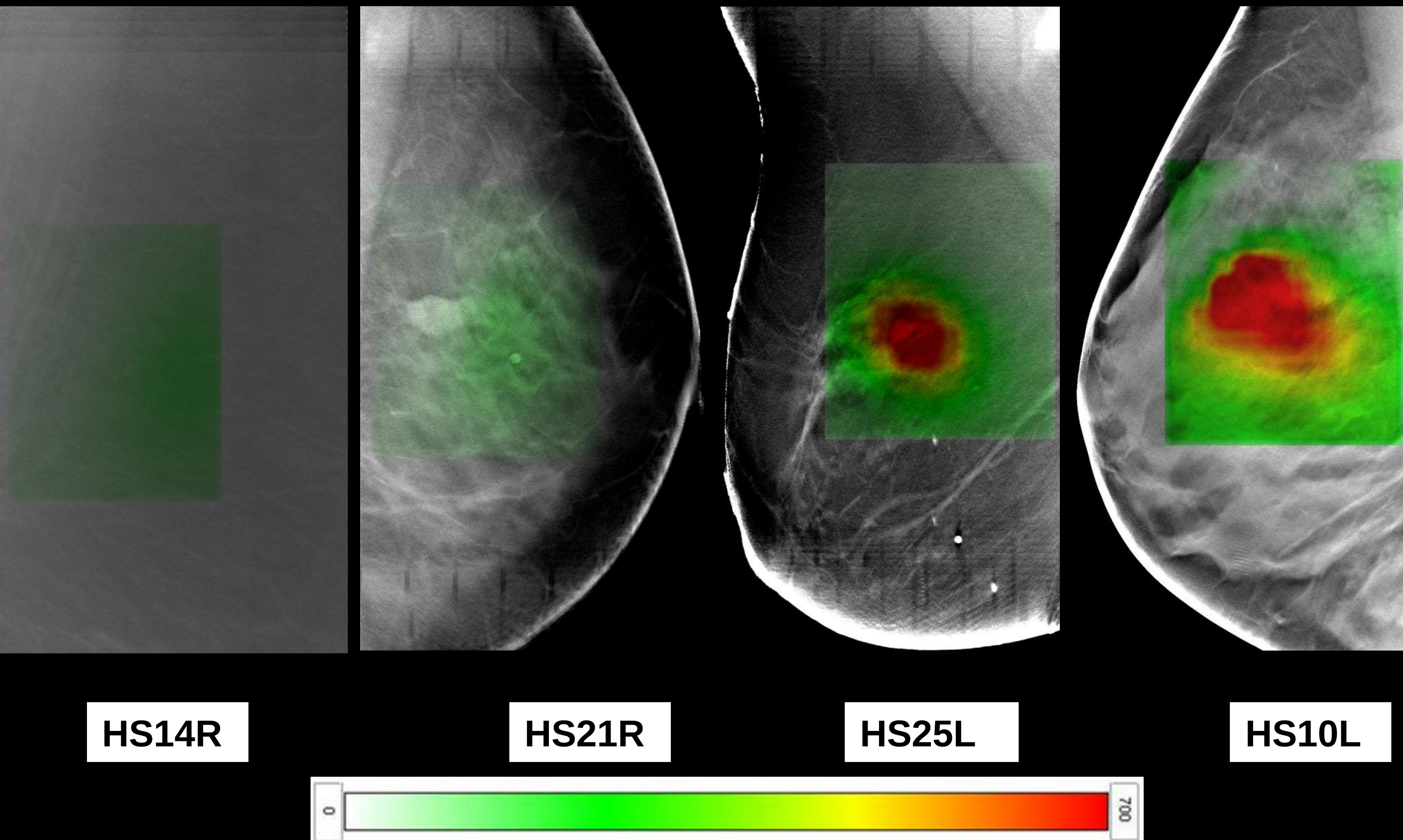
HS14L
LCM=137

HS21L
LCM=328

HS25R
LCM=709

HS10R
LCM=1230

Which ones have cancer?



Can EIT Improve Sensitivity and Specificity in screening for Breast Cancer
?

Questions and Suggestions Happily Received by

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$$\alpha \equiv \sigma + i\omega\varepsilon, \alpha_0 \equiv \sigma_0 + i\omega\varepsilon_0, \beta \equiv -i\omega\mu, \beta_0 \equiv -i\omega\mu_0$$

$$\nabla \wedge H = \alpha E, \qquad \nabla \wedge H_0 = \alpha_0 E_0$$

$$\nabla \wedge E = \beta H, \qquad \nabla \wedge E_0 = \beta_0 H_0$$

$$\nabla \cdot [H \wedge E_0 - H_0 \wedge E]$$

$$= (\alpha - \alpha_0) E \cdot E_0 + (\beta - \beta_0) H \cdot H_0$$

$$\int_S \nu \cdot [H \wedge E_0 - H_0 \wedge E] dS$$

$$= \int_B (\alpha - \alpha_0) E \cdot E_0 + (\beta - \beta_0) H \cdot H_0 \, dx$$