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On determining a conformal-euclidean metric by its copy (Inverse kinematic problem with internal sources)

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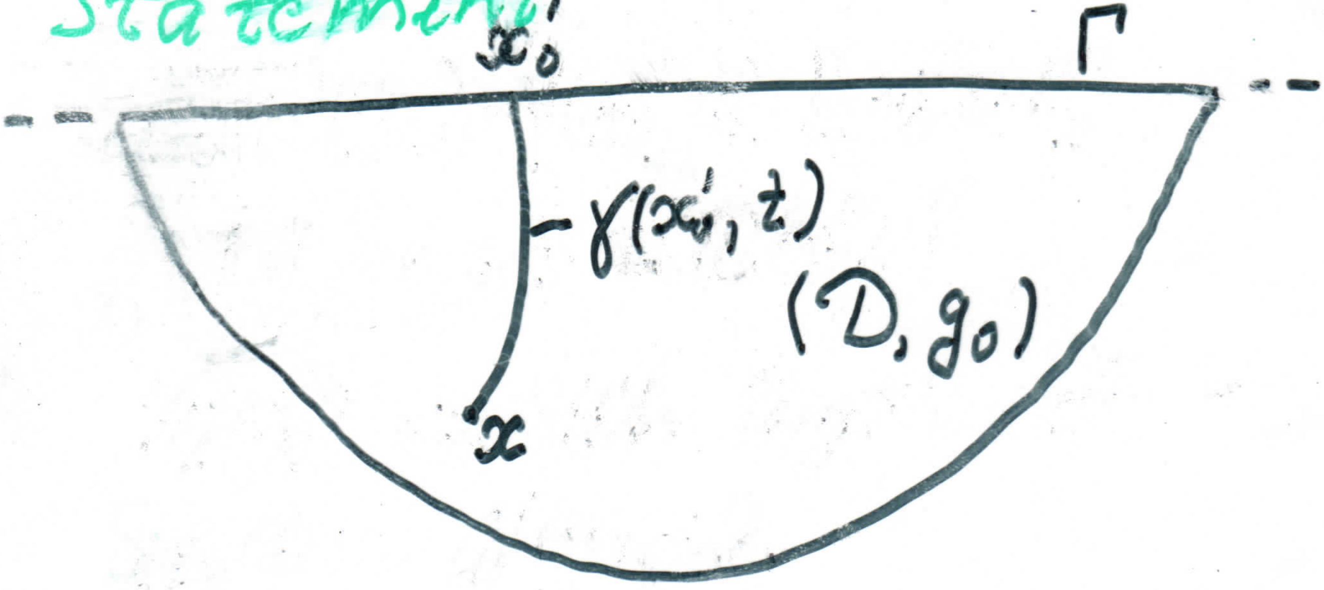
Statement:



$$g^0_{ij}(x) = \delta_{ij} / c^2(x)$$

Inverse data: $\tau(x, x')$, $x \in D$

$x' \in U(x_0') \subset \Gamma$, $c|_\Gamma$

$(c, D) = ?$

②

semigeodesic coordinates of the (·) x:

$x'(x)$, $t(x) = \tau(x', x)$

$x'(x)$ - geodesic projection

$\gamma(x_0', t)$ geodesic, starting from

(·) $x_0' \in \Gamma$ orthogonally to Γ

Assumption: $\gamma : \tilde{D} \to D$,

$$\gamma(x_0', t) = x$$

is diffeomorphism

$$\tilde{D} = \gamma^{-1}(D) = \{(x_0', t) \mid x_0' \in \Gamma, \; 0 \le t < T(x_0')\}$$

$T(x_0')$ is the length of geodesic $\gamma(x_0', t)$.

$\tilde{D}$ is known

$$\gamma(\Gamma) = \Gamma$$

(3)

Metric g

The map γ keeps the distance

$$\tau(x, x') = \tau_g(y, x')$$

where $y = (x', t)$

$$g = \gamma^* g_0$$

Eikonal equation

$$|\nabla \tau_g|^2 = g^{ij}(y) \frac{\partial \tau_g}{\partial y^i} \frac{\partial \tau_g}{\partial y^j} = 1$$

$\Longrightarrow$ g (copy of g_0)

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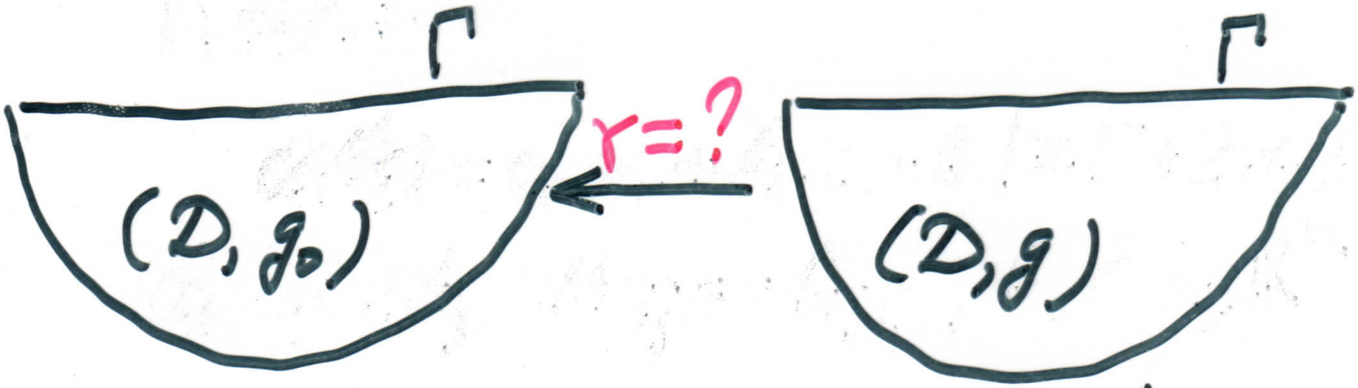
Pullback problem



$g = \gamma^* g_0$

Yamabe problem:

$$a \Delta_g u + R_g u = 0$$

Conformal Killing's vector field (CKVF):

- $$Ku := \sigma \nabla u - \frac{g}{n} \delta u = 0$$

$$\frac{1}{2}\left(\nabla_i u_j + \nabla_j u_i\right) - \frac{g_{ij}}{n} \delta u = 0$$

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$n=2:$ $\nabla u^1 = \nabla_{\perp} u^2$

$n>2:$

$$u(x) = a_0 x + Ax - b|x|^2 + 2x(b,x) + c$$

$a_0 = \text{const}$, $A_{ij} = -A_{ji}$, $b, c \in \mathbb{R}^n$

Solving

1). Take standardt basic vector fields

$$e_{(k)},\ e^i_{(k)} = \delta^i_k,\quad i,k = \overline{1,n}$$

$$K^0 e_{(k)} = \sigma \nabla^0 e_{(k)} - \frac{g^0}{n} \delta^0 e_{(k)} = 0$$

$e_{(k)}$ are CKVF

(6)

2). $u_{(k)} = \gamma^{*} e_{(k)}$

CKVF on $\gamma^{-1}(D)$

$$\sigma \nabla u_{(k)} - \frac{g}{n} \delta u_{(k)} = 0 \qquad (1)$$

$k = 1, \dots, n \qquad g = \gamma^{*} g_0$

$$\begin{cases} u^{i}_{(k)}(x', 0) = \delta^{i}_{k}, \quad k = 1, \dots n-1 \\ u^{i}_{(n)}(x', 0) = \gamma^{i}_{t}(x', 0) = c(x')(0, \dots, 1) \end{cases} \qquad (2)$$

$(1), (2) \Longrightarrow u_{(k)}$

If $n > 2$ dimension of CKVF is finite! $\Longrightarrow$ (1), (2) is stable

3) $u_{(k)}$, $e_{(k)}$ are known

The equality

$$u_{(k)} = \gamma^* e_{(k)}$$

determines Jacobi's matrix of γ. In partiqularly we get

$$\frac{d\gamma^i}{dt}(x_0',t) = u_n^i(x_0',t)$$

$\Rightarrow$

$$\gamma(x_0',t) = x_0' + \int_0^t u_{(n)}(x_0',s)\,ds$$

$$c(\gamma(x_0',t)) = |\gamma_t(x_0',t)|$$

The problem is solved $\quad D=\gamma(\tilde{D})$