Fractal Weyl Laws for asymptotically hyperbolic manifolds

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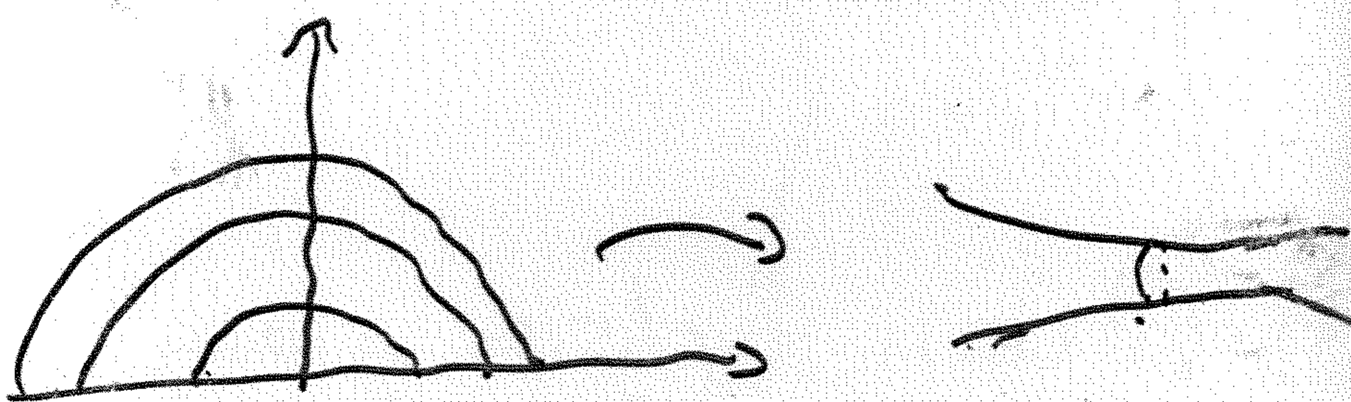
Example 1

Hyperbolic cylinder

$\langle z \mapsto 2z \rangle \backslash \mathbb{H}^2$



$\cong (\mathbb{R} \times S^1, dr^2 + \cosh^2 r \, d\theta^2)$

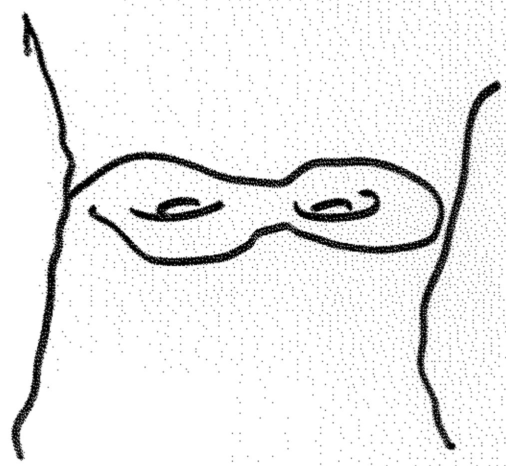
jt with

Semyon Dyatlov

2) Cocompact Fuchsian group

$\Gamma \backslash \mathbb{H}^3 \cong (\mathbb{R} \times \Sigma, dr^2 + \cosh^2 r\, ds)$

(Σ, dS) compact surface



3) Quasi fuchsian group

$\Gamma \backslash \mathbb{H}^3$ diffeo to $\mathbb{R} \times \Sigma$

two hyperbolic structures on Σ

Trapped set $K \subset S^*X$

$K = \{$ geodesics which stay in a compact set for all time $\}$

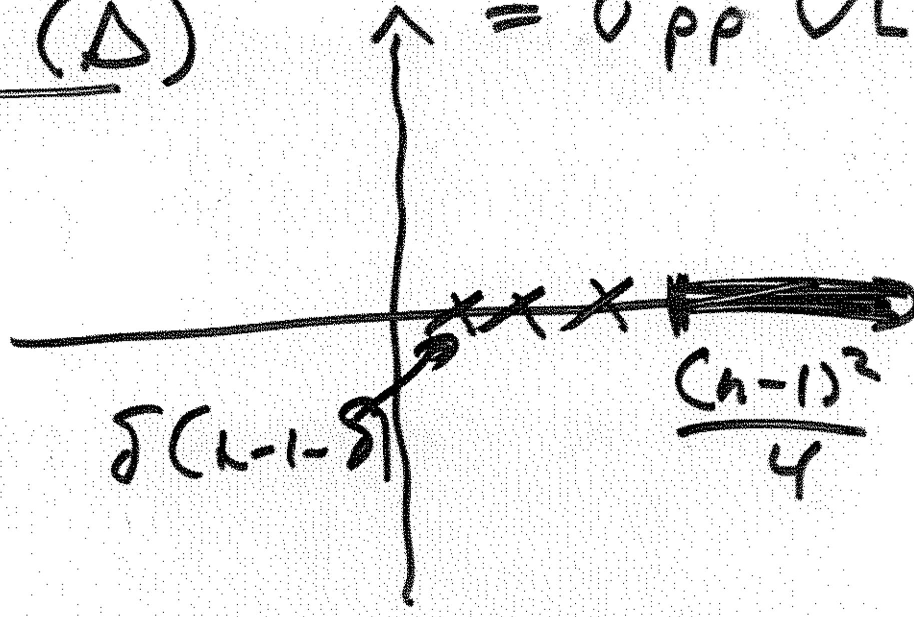
1) K = two copies of S^1
$= S^* S^1$ $\dim K = 1$

2) $K = S^*\Sigma$ $\dim K = 3$

3) K fractal, between the two copies of $S^*\Sigma$
$3 < \dim K < 5$

3

$\underline{\mathrm{Spec}(\Delta)}$ $= \sigma_{pp} \cup [\frac{(n-1)^2}{4}, \infty)$



σ_{pp} finite or empty subset of $(0, \frac{(n-1)^2}{4})$

$\delta = \frac{\dim K - 1}{2}$

If $\delta > \frac{n-1}{2}$ then

$\delta(n-1-\delta)$ is first eigen value

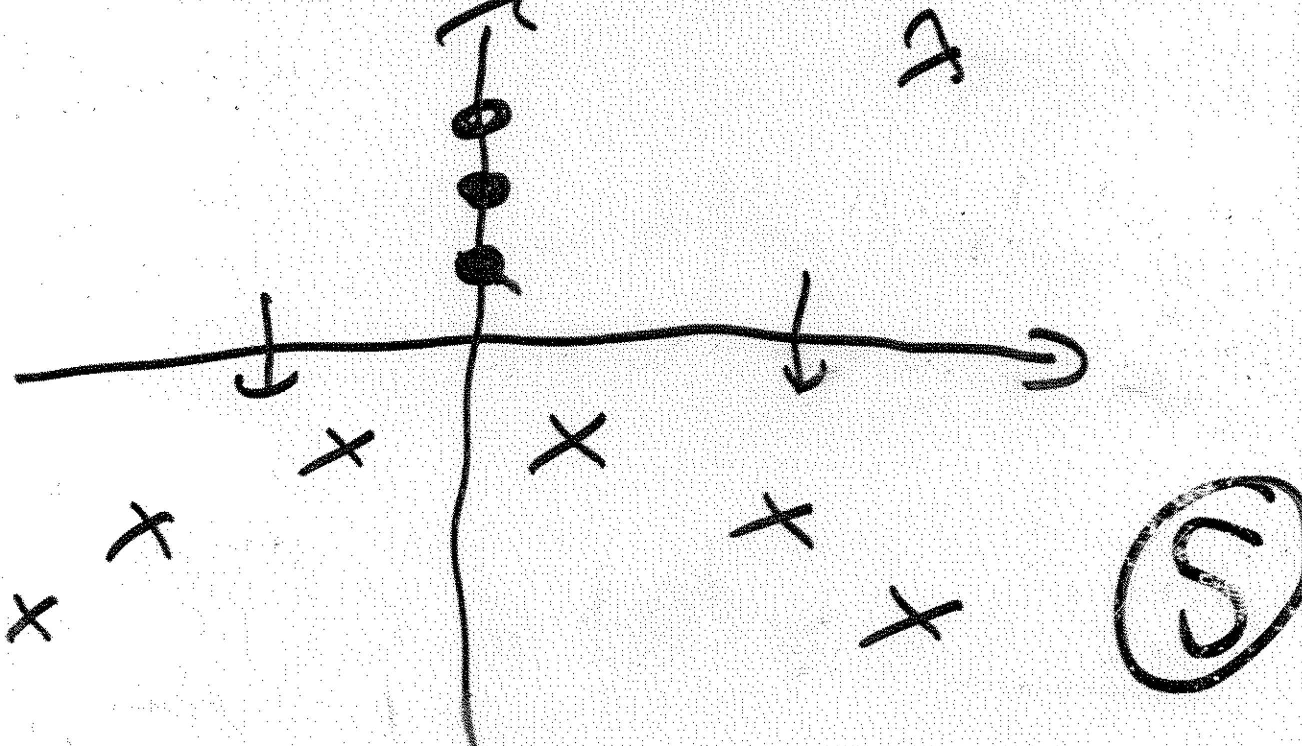
<u>Patterson-Sullivan theory</u>

$$\left(\Delta - \frac{(n-1)^2}{4} - \lambda^2\right)^{-1}$$

meromorphic in $\operatorname{Im}\lambda > 0$

extends (between weighted spaces)

to $\mathbb{C}$



Mazzeo–Melrose, ~~Guillopé–Zw~~

Guillopé–Zworski; Guillarmou,

Vasy

Let $m(\lambda)$ = multiplicity of the resonance at λ

Thm $\forall R \geq 0 \ \exists C > 0 \ \forall t \in \mathbb{R}$

$$\sum_{|\lambda - t| < R} m(\lambda) \leq C(1+|t|)^{\delta}$$

Compare with Weyl Law for compact mflds

$$\sum_{|\lambda - t| < R} m(\lambda) \leq C(1+|t|)^{n-1}$$

"Full trapping" $\dim K = 2n-1$, $\delta = n-1$

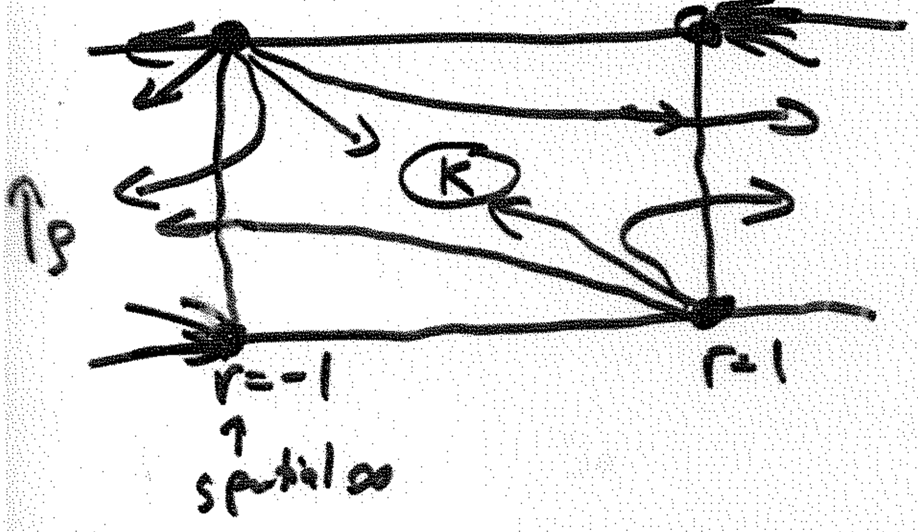
Sjöstrand, Zworski, Guillopé - Lin - Zworski, $\boxed{\text{Sj-Zw}}$

Proof

$$(-1,1)_r \times S^1_\theta$$

ρ dual to r

$$\frac{dr^2}{(1-r^2)^2} + \frac{d\theta^2}{(1-r^2)}$$



← fiber infinity

Let $\mu = (1-r^2)$

Square of [illegible]df

$$\Delta = -\mu^2 \partial_r^2 - \mu \partial_\theta^2 + \text{terms}$$

$P(z) =$ weighted, conjugated version of $h^2\Delta - z$ 7

$$= -\mu h^2 \partial_r^2 - h^2 \partial_\theta^2 + \text{terms}$$

continues smoothly and nondeg.