MA 26100
EXAM 1
02/22/2022
TEST/QUIZ NUMBER: **1010**

NAME ____________________ YOUR TA'S NAME ____________________

STUDENT ID # ____________________ RECITATION TIME ____________________

You must use a #2 pencil on the scantron sheet. Write **1010** in the TEST/QUIZ NUMBER boxes and blacken in the appropriate digits below the boxes. On the scantron sheet, fill in your **TA**'s name for the INSTRUCTOR and **MA 261** for the COURSE number. Fill in whatever fits for your first and last NAME. The STUDENT IDENTIFICATION NUMBER has ten boxes, so use **00** in the first two boxes and your PUID in the remaining eight boxes. Fill in your three-digit SECTION NUMBER. If you do not know your section number, ask your TA. Complete the signature line.

There are **12** questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work in this exam booklet and indicate your answers in the booklet in case the scantron is lost. Use the back of the test pages for scrap paper. Turn in both the scantron sheet and the exam booklet when you are finished.

If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ______________________________________

STUDENT SIGNATURE: ______________________________________

1. Find the plane tangent to the surface $z^2 = 2xy$ at $(1, 2, -2)$.

A. $2x + y - 2z = 8$

B. $x + 2y - 2z = 9$

C. $2x + y + 2z = 0$

D. $4x + 2y - z = 10$

E. $x + 2y + 2z = 1$

F. $4x + 2y + z = 6$

2. Find the absolute maximum value, M, and the absolute minimum value, m, of the function $f(x, y) = x^2 + y^2 - 4y + 4$ on the closed disk $\{(x, y) : x^2 + y^2 \leq 16\}$.

A. $M = 36$ and $m = 0$

B. $M = 36$ and $m = 4$

C. $M = 28$ and $m = 2$

D. $M = 28$ and $m = 0$

E. $M = 28$ and $m = 4$

F. $M = 36$ and $m = 2$

3. Consider the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$. Which of the following statements is true?

A. The limit does not exist, because the path-restricted limit approaching $(0,0)$ along the diagonal $y = x$ does not exist.

B. The limit does not exist, even though the path-restricted limits approaching $(0,0)$ along the x-axis and the y-axis are both 0.

C. The limit does not exist, because the path-restricted limits approaching $(0,0)$ along the x-axis and the y-axis are different.

D. The limit is 0, and the limit along any path approaching $(0,0)$ is also 0.

E. The limit is 0, because the path-restricted limit approaching $(0,0)$ along the diagonal $y = x$ is 0.

F. The limit is 0, even though the path-restricted limits approaching $(0,0)$ along the x-axis and the y-axis are different.

4. Identify the surface that does **not** contain the curve

$$\vec{\boldsymbol{r}}(t) = \langle \cos t, -\cos t, \sin t \rangle$$

A. Plane: $x + y = 0$

B. Circular cylinder: $y^2 + z^2 = 1$

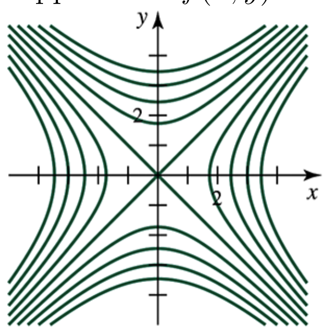
C. Ellipsoid: $\dfrac{x^2}{2} + \dfrac{y^2}{2} + z^2 = 1$

D. Circular cylinder: $x^2 + y^2 = 1$

E. Ellipsoid: $\dfrac{x^2}{3} + \dfrac{2y^2}{3} + z^2 = 1$

F. Circular cylinder: $x^2 + z^2 = 1$

5. Suppose $z = f(x, y)$ has the following level curves:



The surface formed by the graph of f could be which of the following?

A. Plane

B. Hyperbolic paraboloid

C. Hyperboloid of two sheets

D. Ellipsoid

E. Elliptic paraboloid

F. Elliptic cone

6. A ball is launched from an initial location of $(0, h)$, with initial velocity vector $\langle 10, 10\rangle$. Use the constant $g > 0$ for the acceleration due to gravity, and assume the gravitational force points in the direction of the negative y-axis. Determine the location of the ball when it is at its maximum height.

A. $\left(\dfrac{50 + 5\sqrt{100 + 2gh}}{g}, \dfrac{10 + \sqrt{100 + 2gh}}{g}\right)$

B. $\left(\dfrac{100}{g}, \dfrac{10 + \sqrt{100 + 2gh}}{g}\right)$

C. $\left(\dfrac{50 + 5\sqrt{100 + 2gh}}{g}, \dfrac{100 + 3gh + 10\sqrt{100 + 2gh}}{4g}\right)$

D. $\left(\dfrac{100}{g}, \dfrac{50 + gh}{g}\right)$

E. $\left(\dfrac{100}{g}, \dfrac{100 + 3gh + 10\sqrt{100 + 2gh}}{4g}\right)$

F. $\left(\dfrac{50 + 5\sqrt{100 + 2gh}}{g}, \dfrac{50 + gh}{g}\right)$

7. The line ℓ passes through the points $(1,1,1)$ and $(2,0,1)$. The plane Q contains the points $(0,0,0)$, $(1,2,2)$, and $(1,0,1)$. Find the intersection point of ℓ and Q.

A. $(0,2,1)$

B. $(-1,4,1)$

C. $(2,-2,1)$

D. $(-2,4,1)$

E. $(4,-2,1)$

F. $(3,-1,1)$

8. Find the arclength function for

$$\vec{\boldsymbol{r}}(t) = \langle \cos t + t\sin t,\ \sin t - t\cos t,\ t^2 \rangle,$$

giving the length of the curve measured from $(1,0,0)$ in the direction of positive orientation.

A. $s(t) = \dfrac{1}{3}(1+t^2)^{3/2} - \dfrac{1}{3}$

B. $s(t) = \displaystyle\int_0^t \sqrt{1+u^2+u^4}\,du$

C. $s(t) = \dfrac{\sqrt{5}}{2}t^2$

D. $s(t) = \sqrt{5}\,t$

E. $s(t) = \dfrac{2\sqrt{2}}{3}(1+t)^{3/2} - \dfrac{2\sqrt{2}}{3}$

F. $s(t) = \dfrac{t\sqrt{1+t^2} + \ln|t+\sqrt{1+t^2}|}{2}$

9. Compute the directional derivative of $f(x,y,z) = x^2y + yz^2$ at $(1,1,1)$ in the direction $\frac{2}{3}\vec{\boldsymbol{i}} + \frac{1}{3}\vec{\boldsymbol{j}} + \frac{2}{3}\vec{\boldsymbol{k}}$.

A. 3

B. $\frac{8}{3}$

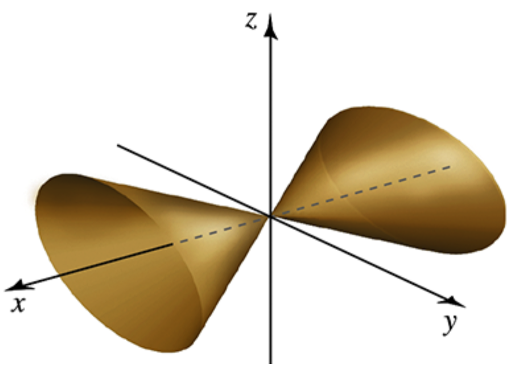
C. 8

D. 2

E. 10

F. $\frac{10}{3}$

10. Which of these equations has a graph like the pictured elliptic cone, with vertex at the origin and opening in the direction of the x-axis.



A. $y^2 - 4z^2 - 16x^2 = 1$

B. $y^2 + 4z^2 - 16x^2 = 1$

C. $y^2 + 4z^2 + 16x^2 = 1$

D. $y^2 - 4z^2 + 16x^2 = 0$

E. $y^2 + 4z^2 - 16x^2 = 0$

F. $y^2 - 4z^2 + 16x^2 = 1$

11. Classify all critical points of $f(x, y) = \dfrac{x^3}{3} - \dfrac{y^3}{3} + 2xy$, and choose the correct summary from the answer choices below.

A. Two local maximums.

B. One local maximum and one saddle point.

C. One local maximum and one local minimum.

D. One local minimum and one saddle point.

E. Two local minimums.

F. Two saddle points.

12. Suppose f is a function of x, y, and z, with $f_x(1, 1, 1) = 1$, $f_y(1, 1, 1) = 2$, and $f_z(1, 1, 1) = 3$. If $x = x(t) = t^2$, $y = y(t) = t^3$, and $z = z(t) = t^4$, find $\dfrac{df}{dt}$ when $t = 1$.

A. 54

B. 6

C. 20

D. 144

E. 9

F. 24