

MA 26100
EXAM 2 Form 01
April 1, 2019

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes):

01

You must use a #2 pencil on the mark-sense sheet (answer sheet). On the mark-sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark-sense sheet.

There are 10 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1-10. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 6:50. If you don't finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. The extreme values of $f(x, y, z) = 3x + 2y + 6z$ with constraint $x^2 + y^2 + z^2 = 4$ are

- A. The maximum of f is 7 and the minimum of f is -14
- B. The maximum of f is 14 and the minimum of f is -14
- C. The maximum of f is 7 and the minimum of f is -7
- D. The maximum of f is 14 and the minimum of f is -7
- E. The maximum of f is 28 and the minimum of f is -28

We consider $g(x, y, z) = x^2 + y^2 + z^2 - 4$. Then

$$\nabla f = \langle 3, 2, 6 \rangle, \quad \nabla g = \langle 2x, 2y, 2z \rangle.$$

Hence $\lambda \nabla f = \nabla g \Leftrightarrow 3\lambda = 2x, 2\lambda = 2y, 6\lambda = 2z$

We get $y = \lambda, x = \frac{3}{2}\lambda, z = 3\lambda$. Now we find λ s.t. $g(x, y, z) = 0$:

$$\left(\frac{3}{2}\lambda\right)^2 + \lambda^2 + (3\lambda)^2 = 4 \Leftrightarrow \left(\frac{9}{4} + 10\right)\lambda^2 = 4$$

$$\Leftrightarrow \lambda^2 = \frac{16}{49} \Leftrightarrow \lambda = \pm \frac{4}{7}$$

We get two critical points: $\pm \left(\frac{6}{7}, \frac{4}{7}, \frac{12}{7}\right)$

Then $f\left(\frac{6}{7}, \frac{4}{7}, \frac{12}{7}\right) = 14$
 $f\left(-\frac{6}{7}, -\frac{4}{7}, -\frac{12}{7}\right) = -14$

(B)

2. Reverse the order of integration and evaluate the double integral

$$I = \int_0^1 \int_{x^2}^1 6\sqrt{y} \cos y^2 dy dx.$$

A. $\sin 1$

B. $2 \sin 1$

C. $3 \sin 1$

D. $3 \cos 1$

E. $2 \cos 1 - 2$

Domain:

$$D = \{0 \leq x \leq 1, x^2 \leq y \leq 1\}$$

$$= \{0 \leq y \leq 1, 0 \leq x \leq \sqrt{y}\}$$

② - Ctd. Thus if we change the order of integration we get

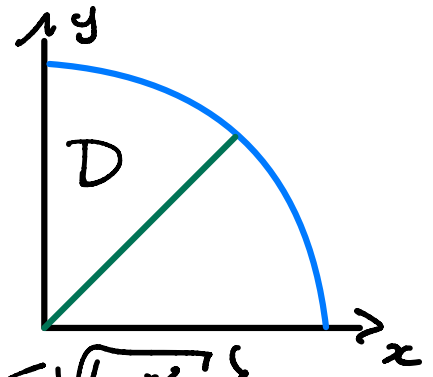
$$\begin{aligned} I &= 6 \int_0^1 \int_0^{\sqrt{y}} \sqrt{y} \cos y^2 \, dx \, dy \\ &= 6 \int_0^1 y \cos y^2 \, dy \\ &= 3 \int_0^1 \sin y^2 \, dy \end{aligned}$$

Thus $I = 3 \operatorname{si} 1$ ©

3. Evaluate

$$I \equiv \int_0^{\frac{1}{\sqrt{2}}} \int_x^{\sqrt{1-x^2}} 3\sqrt{x^2+y^2} dy dx$$

using polar coordinates.



$$\begin{aligned} \text{Domain } D &= \left\{ 0 \leq x \leq \frac{1}{\sqrt{2}}, x \leq y \leq \sqrt{1-x^2} \right\} \\ &= \left\{ 0 \leq r \leq 1, \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} \right\} \end{aligned}$$

- A. $\frac{\pi}{2}$
- B. $\frac{\pi}{3}$
- C. $\frac{\pi}{6}$
- D. $\frac{\pi}{4}$
- E. $\frac{\pi}{12}$

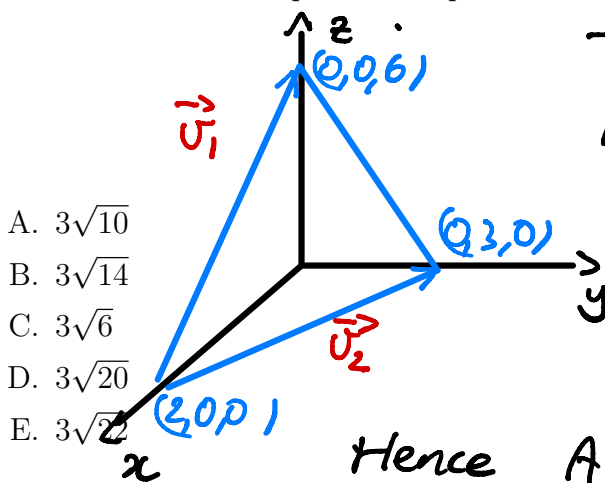
$$\begin{aligned} \text{Then} \\ I &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 3r \cdot r dr d\theta \\ &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 3r^2 dr d\theta \end{aligned}$$

Thus

$$I = \frac{\pi}{4}$$

(D)

4. Find the area of the part of the plane $3x + 2y + z = 6$ that is in the first octant.



- A. $3\sqrt{10}$
- B. $3\sqrt{14}$
- C. $3\sqrt{6}$
- D. $3\sqrt{20}$
- E. $3\sqrt{2}$

$$A = 3\sqrt{14}$$

The area is given by

$$A = \frac{1}{2} |\vec{u}_1 \times \vec{u}_2|. \text{ Moreover}$$

$$\vec{u}_1 \times \vec{u}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 6 \\ -2 & 3 & 0 \end{vmatrix}$$

$$= -6 < 3, 2, 1 >$$

$$A = \frac{1}{2} \times 6 \times \sqrt{9+4+1} =$$

4

(B)

5. Consider the tetrahedron E with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$. Express

$$\mathcal{Q} = \iiint_E x \, dV$$

as an iterated integral in the order $dz \, dy \, dx$.

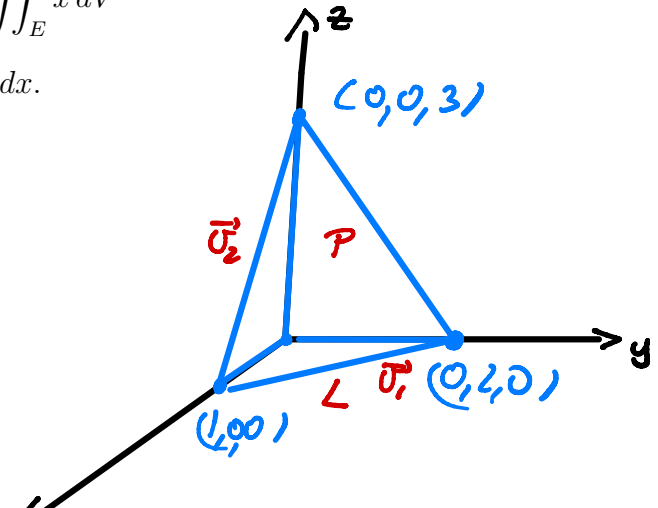
A. $\int_0^1 \int_0^{2-2x} \int_0^{-3x-\frac{3}{2}y-3} x \, dz \, dy \, dx$

B. $\int_0^1 \int_0^{2-2x} \int_0^{-3x+\frac{3}{2}y+3} x \, dz \, dy \, dx$

C. $\int_0^1 \int_0^{2-2x} \int_0^{-3x-\frac{3}{2}y+3} x \, dz \, dy \, dx$

D. $\int_0^1 \int_0^{2-2x} \int_0^{3x+\frac{3}{2}y-3} x \, dz \, dy \, dx$

E. $\int_0^1 \int_0^{2-2x} \int_0^{3x-\frac{3}{2}y-3} x \, dz \, dy \, dx$



In xy -plane the line L is given by $L: y = 2 - 2x$.

Then the plane P has a normal given by $\vec{v}_1 \times \vec{v}_2 = \langle 6, 3, 2 \rangle$
 Thus $P: 6x + 3y + 2z = 6 \Leftrightarrow z = 3 - \frac{3}{2}y - 3x$. We get

$$\mathcal{Q} = \int_0^1 \int_0^{2-2x} \int_0^{-3x-\frac{3}{2}y+3} x \, dz \, dy \, dx$$

(C)

6. The triple integral

$$J = \int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} 8(x^2 + y^2) \, dz \, dy \, dx$$

when converted to cylindrical coordinates becomes

The domain in xy -plane is

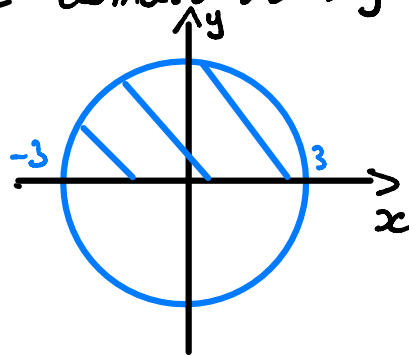
A. $\int_0^\pi \int_0^3 \int_0^r 8r^3 \, dz \, dr \, d\theta$

B. $\int_0^\pi \int_0^3 \int_0^r 8r^3 z \, dz \, dr \, d\theta$

C. $\int_0^\pi \int_0^9 \int_0^r 8r^2 \, dz \, dr \, d\theta$

D. $\int_0^\pi \int_0^3 \int_0^r 8r^2 z \, dz \, dr \, d\theta$

E. $\int_0^\pi \int_0^9 \int_0^r 8r^2 \, dz \, dr \, d\theta$



Thus $D = \{0 \leq r \leq 3, 0 \leq \theta \leq \pi, 0 \leq z \leq r\}$

We get $J = \int_0^\pi \int_0^3 \int_0^r 8r^2 z \, dz \, dr \, d\theta$

$$J = \int_0^\pi \int_0^3 \int_0^r 8r^2 z \, dz \, dr \, d\theta$$

(A)

$$\left\langle 1 \leq \rho \leq 2, 0 \leq \varphi \leq \frac{\pi}{2}, 0 \leq \theta \leq \frac{\pi}{2} \right\rangle$$

J

7. Evaluate the triple integral $\iiint_E (x^2 + y^2) dV$ where E is the solid region in the first octant which is outside the sphere $x^2 + y^2 + z^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 4$.

In spherical coordinates, the domain is

Then $x^2 + y^2 = (\rho \sin \varphi \cos \theta)^2 + (\rho \sin \varphi \sin \theta)^2$

$= \rho^2 \sin^2 \varphi$. Hence we get

A. $\frac{8\pi}{15}$
 B. $\frac{9}{16}\pi$
 C. $\frac{24}{5}\pi$
 D. $\frac{63}{30}\pi$
 E. $\frac{31}{15}\pi$

$$J = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_1^2 \rho^2 \sin^2 \varphi \cdot \rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} (1 - \cos^2 \varphi) \sin \varphi \, d\varphi \int_1^2 \rho^4 \, d\rho$$

$$= \frac{\pi}{2} \times \left(-\left(\cos \varphi - \frac{1}{3} \cos^3 \varphi \right) \Big|_0^{\frac{\pi}{2}} \right) \times \frac{1}{5} (2^5 - 1^5)$$

$$= \frac{\pi}{2} \times \frac{2}{3} \times \frac{31}{5}$$

We get

$$J = \frac{31\pi}{15}$$

(E)

8. Let $f(x, y, z) = x^2 + y^3 + z^4$ and $g(x, y, z) = 3x + 4y + z^2/2$. If $\vec{\nabla} f(2, 1, -1)$ is perpendicular to $\vec{\nabla} g(a, b, c)$, then

We have $\vec{\nabla} f = \langle 2x, 3y^2, 4z^3 \rangle$,

so that $\vec{\nabla} f(2, 1, -1) = \langle 4, 3, -4 \rangle$

Next $\vec{\nabla} g = \langle 3, 4, c \rangle$, and

- A. $c = 2$
 B. $c = 4$
 C. $c = 6$
 D. $c = 8$
 E. $c = 10$

$$\vec{\nabla} f(2, 1, -1) \cdot \vec{\nabla} g = 0$$

$$\Leftrightarrow 24 - 4c \Leftrightarrow c = 6$$

(C)

9. Evaluate the line integral $\int_C xy dx - y^2 dy$, where C is the line segment from $(0,0)$ to $(2,6)$.

Here $\vec{F} = \langle xy, -y^2 \rangle$, which is not conservative.
Then C is parametrized as $\vec{r}(t) = \langle 2t, 6t \rangle$ for $0 \leq t \leq 1$. Hence the line integral is

$$\int_0^1 \langle 2t \times 6t, -(6t)^2 \rangle \cdot \langle 2, 6 \rangle dt$$

$$= \int_0^1 (24t^2 - 6^3 t^2) dt$$

- A. 42
B. -36
C. 36
D. -64
E. -44

We get $J = -64$ D

10. Evaluate the line integral $\int_C \frac{9x}{y} ds$, where C is the curve $x = \frac{t^3}{3}$, $y = \frac{t^4}{4}$ with $1 \leq t \leq 2$.

We have $\vec{r}(t) = \langle t^2, t^3 \rangle$, so that

$$|\vec{r}'(t)| = (t^4 + t^6)^{\frac{1}{2}} = t^2 (1 + t^2)^{\frac{1}{2}}$$

$$I = \int_1^2 9 \frac{t^3/3}{t^4/4} t^2 (1 + t^2)^{\frac{1}{2}} dt$$

- A. $15^{3/2} - 3^{3/2}$
B. $4(5^{3/2} - 2^{3/2})$
C. $15^{3/2} - 3^{3/2}$
D. $\frac{1}{2}(15^{3/2} - 3^{3/2})$
E. $14^{3/2} - 3^{3/2}$

$$= 12 \int_1^2 t (1 + t^2)^{\frac{1}{2}} dt$$

$$= 6 \int_1^2 (2t) (1 + t^2)^{\frac{1}{2}} dt = 4 (1 + t^2)^{\frac{3}{2}} \Big|_1^2$$

$$I = 4(5^{3/2} - 2^{3/2})$$

B