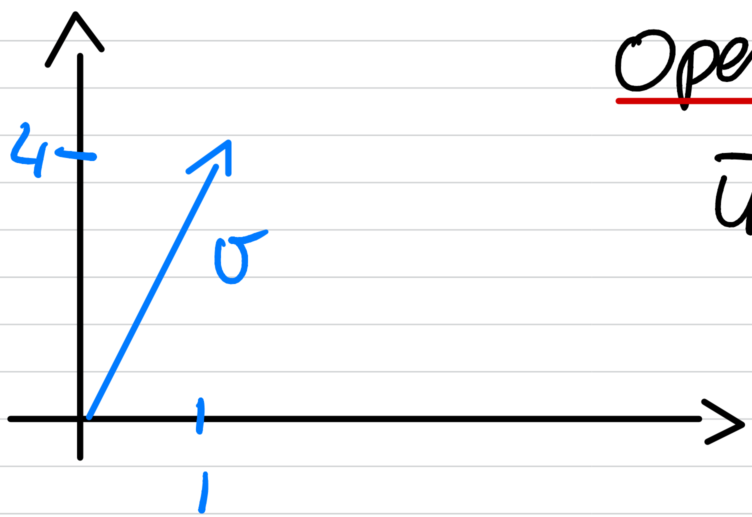
Summary of 1st 3 sections

There are some objects in $\mathbb{R}^2$ and $\mathbb{R}^3$ called vectors

Notation : $\vec{u} = \langle 1, 2, 3 \rangle \in \mathbb{R}^3$

$\vec{w} = \langle 2, 5, -7 \rangle$ $\quad \vec{v} = \langle 1, 4 \rangle \in \mathbb{R}^2$



Operations

$\vec{u} + \vec{w}$

$4\vec{u}$

$-\vec{w} + 3\vec{u}$

Outline

Definition of dot product

Definition 3.

Let

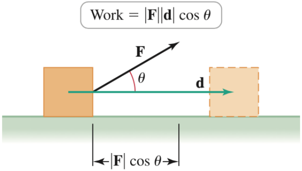
- $\mathbf{u}, \mathbf{v}$ vectors in $\mathbb{R}^3$
- $\theta \in [0, \pi]$ angle between $\mathbf{u}$ and $\mathbf{v}$

Then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}|\,\cos(\theta) \quad \in \mathbb{R}$$

$\in \mathbb{R}^2$ or $\mathbb{R}^3$

Motivation: Work of a force



Analytic expression for the dot product

Theorem 4.

Let

- $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$
- $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$

Then we have

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

Example of dot product

$u \cdot v = 1 \times 4 + 2 \times 5 + 3 \times 6 = 32$

Computation of dot product: If

$$\mathbf{u} = \langle 1, 2, 3 \rangle, \quad \mathbf{v} = \langle 4, 5, 6 \rangle,$$

then according to Theorem 4,

$|u| = \sqrt{1^2 + 2^2 + 3^2}$

$$\mathbf{u} \cdot \mathbf{v} = 32$$

Angle between **u** and **v**: According to Definition 7

$u \cdot v = |u|\,|v| \cos(\theta) \Rightarrow$

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|} = \frac{32}{\sqrt{14 \times 77}}.$$

Thus

$$\theta \simeq 13^{\circ}$$

Outline

Definition of cross product

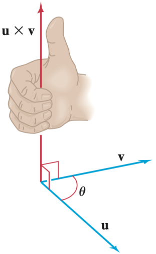
Definition 7.

Let

- **u**, **v** vectors in $\mathbb{R}^3$, with angle $\theta \in [0, \pi]$

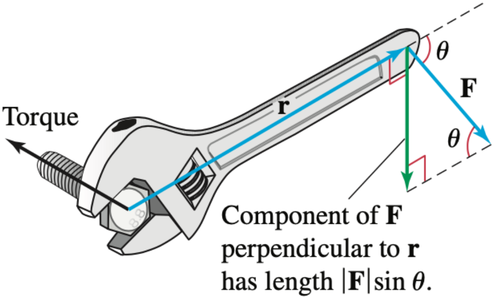
Then $\mathbf{u} \times \mathbf{v}$ is a vector such that

1. Magnitude is $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\,\sin(\theta)$.
2. Direction: given by right hand rule.

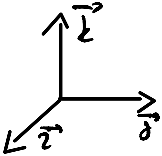


Cross product: illustration

Motivation: Torque



Formula for cross product



Formula: We have

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Example of cross product: If

$$\mathbf{u} = \langle 2, 1, 1 \rangle, \qquad \mathbf{v} = \langle 5, 0, 1 \rangle,$$

then

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & 1 & 1 \\ 5 & 0 & 1 \end{vmatrix} = \langle 1, 3, -5 \rangle$$