Outline

Definition of vector field

Multivariate function: Recall that

- $z = f(x, y)$ was a function of 2 variables
- For each (x, y), $z \in \mathbb{R}$
- This is called a scalar field

Vector field in $\mathbb{R}^2$:

- Of the form $\mathbf{F}(x, y) = \langle f(x, y), g(x, y) \rangle$
- For each (x, y), $\mathbf{F} \in \mathbb{R}^2$, namely $\mathbf{F}$ is a vector

$\vec{F}(x,y)$

Example of vector field

If $x=1, y=1$, then
$\vec{F}(x,y) = \langle x,y \rangle = \langle 1,1 \rangle$
If $x=-3$, $y=4$, then
$\vec{F}(-3,4) = \langle -3,4 \rangle$

Definition of the vector field:

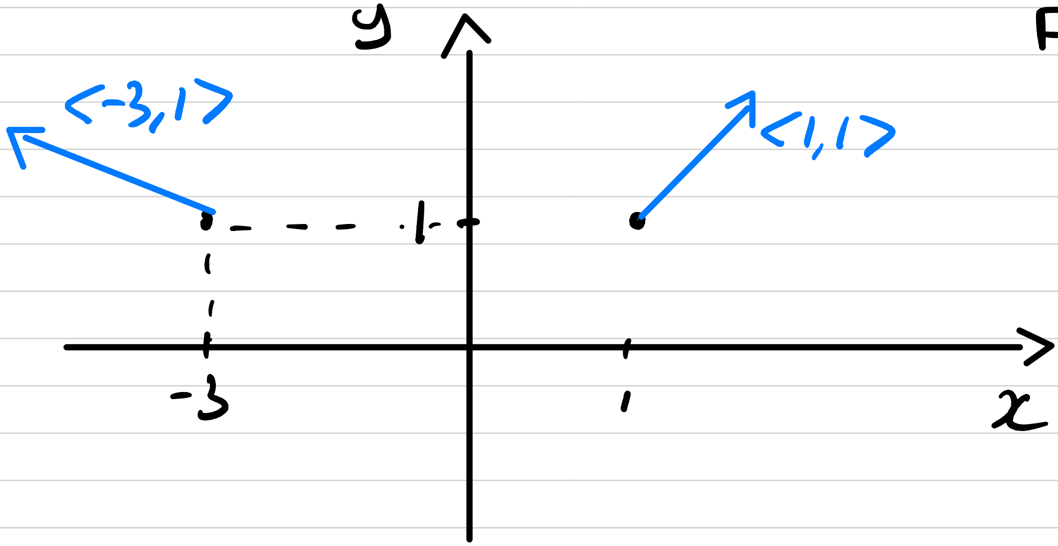
$$\mathbf{F}(x, y) = \langle x, y \rangle$$

Examples of values:

$$\begin{aligned} \mathbf{F}(1,1) &= \langle 1,1 \rangle \\ \mathbf{F}(0,2) &= \langle 0,2 \rangle \\ \mathbf{F}(-1,-2) &= \langle -1,-2 \rangle \end{aligned}$$

Graph

$\vec{F}(x,y) = \langle x, y \rangle$



Shear field (1)

Definition of the vector field:

$$\mathbf{F}(x, y) = \langle 0, x \rangle$$

Problem:

Give a representation of **F**

$\vec{F}(x,y) = \langle 0, x \rangle$

Information on $\vec{F}$

- x- component of $\vec{F}$ is 0

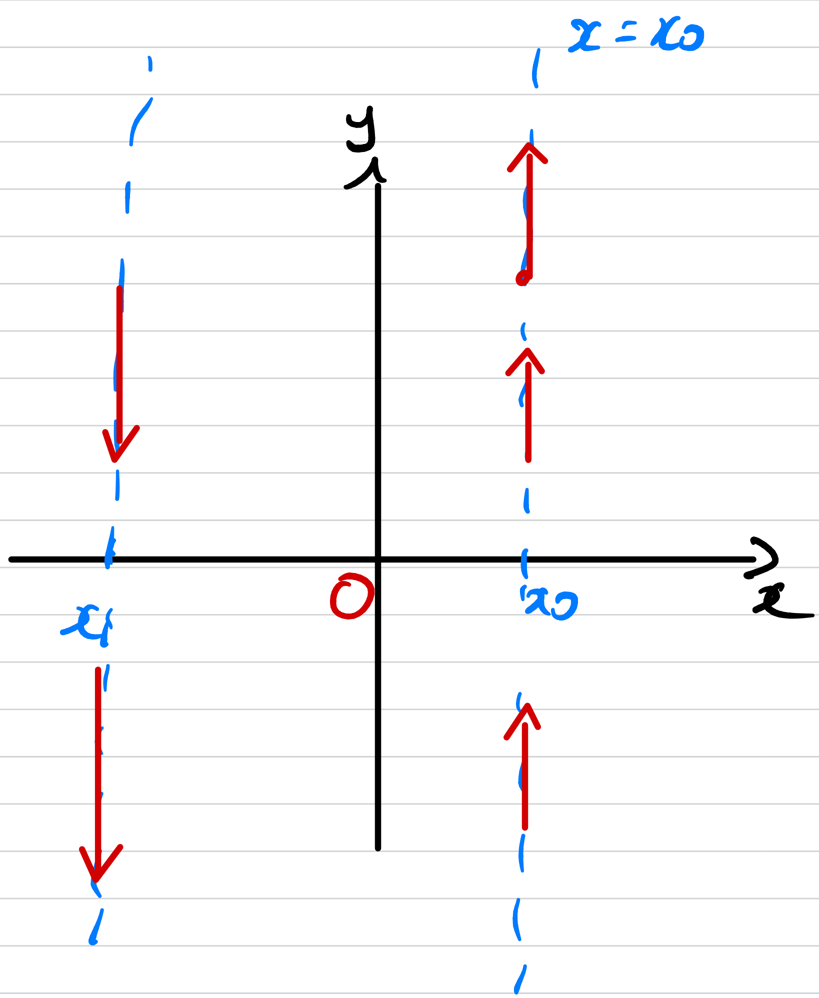
$\Rightarrow$ $\vec{F}(x,y)$ is vertical

- $x > 0 \Rightarrow \vec{F}(x,y)$ upward
- $x < 0 \Rightarrow \vec{F}(x,y)$ downward

- $\vec{F}(x,y)$ does not depend on $y \Rightarrow \vec{F}$ constant along line $x = x_0$

- $|\vec{F}(x,y)| = |x|$

$\Rightarrow$ magnitude longer when we are away from O



Shear field (2)

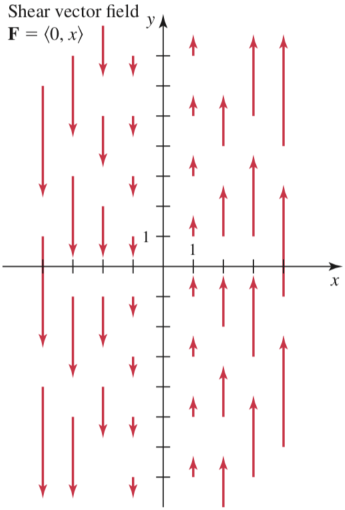
Recall:

$$\mathbf{F}(x, y) = \langle 0, x \rangle$$

Information about the vector field:

1. $\mathbf{F}(x, y)$ independent of y
2. $\mathbf{F}(x, y)$ points in the y direction
3. If $x > 0$, $\mathbf{F}(x, y)$ points upward
4. If $x < 0$, $\mathbf{F}(x, y)$ points downward
5. Magnitude of $\mathbf{F}(x, y)$ gets larger
   $\hookrightarrow$ as we move away from the origin

Shear field (3)



Rotation field (1)

Definition fo the vector field:

$$\mathbf{F}(x, y) = \langle -y, x \rangle$$

Problem:

Give a representation of **F**

Examples

$F(1,2) = \langle -2, 1 \rangle$

$F(3,-4) = \langle 4, 3 \rangle$

Field: $\vec{F}(x,y) = \langle -y, x \rangle$

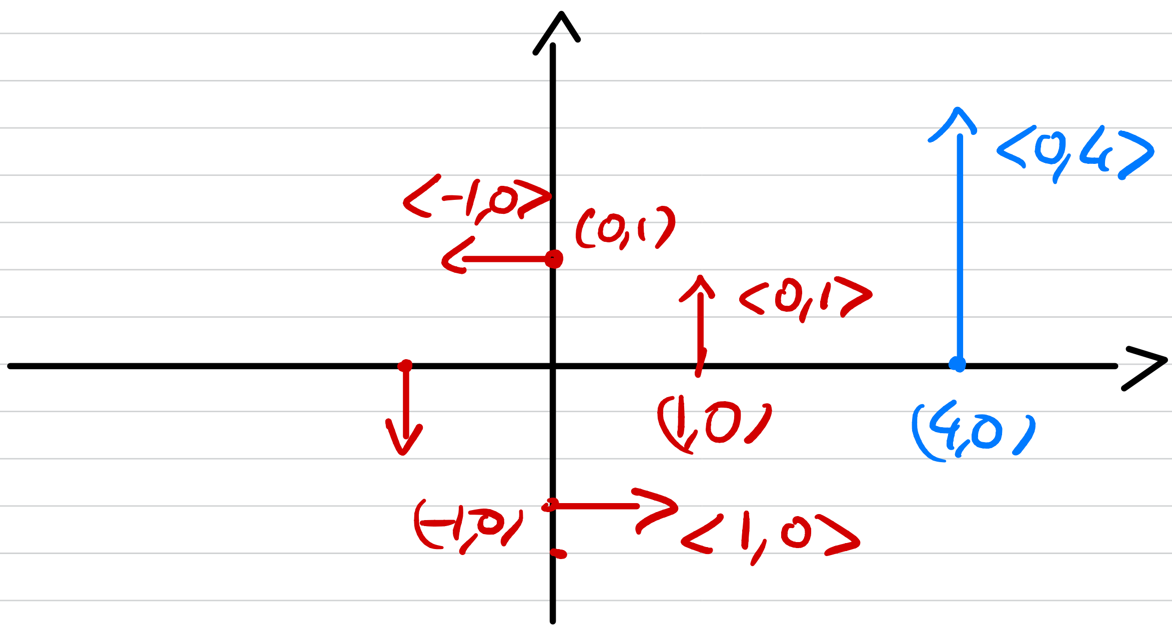
$\vec{F}(1,0) = \langle 0,1 \rangle$

$\vec{F}(0,1) = \langle -1,0 \rangle$

$\vec{F}(-1,0) = \langle 0,-1 \rangle$

$\vec{F}(0,-1) = \langle 1,0 \rangle$

Graph



Claim $\vec{F}(x,y) \perp \langle x,y \rangle$

$$\vec{F}(x,y) \cdot \langle x,y \rangle = \langle -y, x \rangle \cdot \langle x,y \rangle$$

$$= -yx + xy = 0$$

$$\Rightarrow \quad \vec{F}(x,y) \perp \langle x,y \rangle$$

Rotation field (2)

Recall:

$$\mathbf{F}(x, y) = \langle -y, x \rangle$$

Information about the vector field:

1. Magnitude increases as $x \to \infty$ or $y \to \infty$
2. If $y = 0$ and $x > 0$, $\mathbf{F}(x, y)$ points upward
3. If $y = 0$ and $x < 0$, $\mathbf{F}(x, y)$ points downward
4. If $x = 0$ and $y > 0$, $\mathbf{F}(x, y)$ points in negative x direction
5. If $x = 0$ and $y < 0$, $\mathbf{F}(x, y)$ points in positive x direction
6. Draw a few more points
   $\hookrightarrow$ We get a rotation field

Rotation field (3)