Vectors and the geometry of space

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Multivariate calculus - MA 261

Mostly taken from *Calculus, Early Transcendentals*
by Briggs - Cochran - Gillett - Schulz

Outline

1. Vectors in the plane
2. Vectors in three dimensions
3. Dot product
4. Cross product
5. Lines and planes in space
6. Quadric surfaces

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Definition of vectors

Definition 1.

Consider 2 points in the plane

- $P = (x_1, y_1)$
- $Q = (x_2, y_2)$.

Then $\mathbf{u} = \vec{PQ}$ is defined by

$$\mathbf{u} = \vec{PQ} = \langle x_2 - x_1, \, y_2 - y_1 \rangle .$$

Example of vector

Example: Take

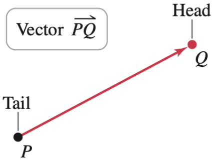
- $P = (-1, 7)$
- $Q = (3, 0)$.

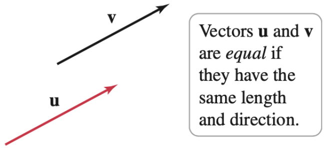
Then

$$\vec{PQ} = \langle 4, -7 \rangle = 4\,\vec{\imath} - 7\,\vec{\jmath}.$$

Opposite of a vector: We have

$$\vec{QP} = \langle -4, 7 \rangle = -4\,\vec{\imath} + 7\,\vec{\jmath} = -\vec{PQ}.$$





Magnitude of a vector

Magnitude: Consider the vector

$$\mathbf{u} = \langle x, y \rangle = x\,\vec{\imath} + y\,\vec{\jmath}.$$

Then the magnitude of **u** is

$$|\mathbf{u}| = \sqrt{x^2 + y^2}$$

Example in $\mathbb{R}^2$: We have

$$\mathbf{u} = \langle 1, 2 \rangle \quad \Longrightarrow \quad |\mathbf{u}| = \sqrt{5}.$$

Example in $\mathbb{R}^3$: We have

$$\mathbf{u} = \langle 1, 2, 3 \rangle \quad \Longrightarrow \quad |\mathbf{u}| = \sqrt{14}.$$

Addition and multiplication of vectors

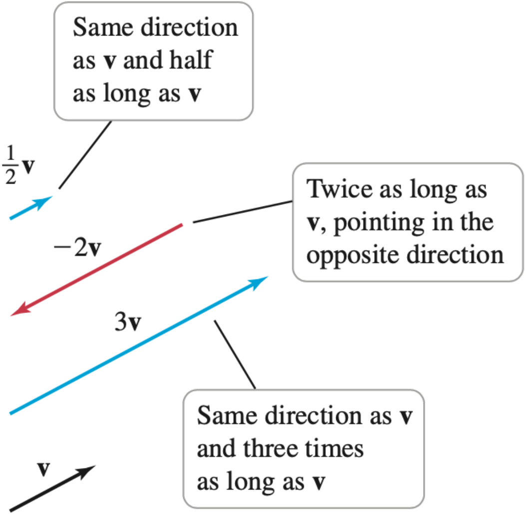
Example: If

$$\mathbf{u} = \langle 1, 2, 3 \rangle \quad \mathbf{v} = \langle 4, 5, 6 \rangle ,$$

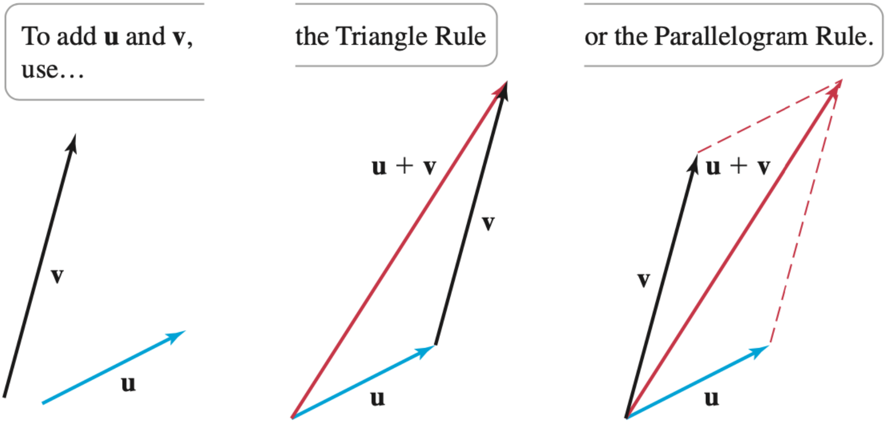
then

$$\begin{aligned} 3\,\mathbf{u} &= \langle 3, 6, 9 \rangle \\ 2\,\mathbf{u} - 3\,\mathbf{v} &= \langle -10, -11, -12 \rangle \end{aligned}$$

Multiplication: geometric interpretation



Addition: geometric interpretation



Unit vectors

Definition 2.

A vector **u** is a unit vector if it has length 1:

$$|u| = 1$$

Examples of unit vectors

Counterexample: Take $\mathbf{u} = (1, 2, 3)$. Then

$$|u| = \sqrt{14} \quad \Longrightarrow \quad u \text{ not unit}$$

Example: Take

$$\mathbf{v} = \frac{1}{\sqrt{14}} \mathbf{u} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

Then $\mathbf{v}$ is unit.

Speed of boat in current

Situation: Assume:

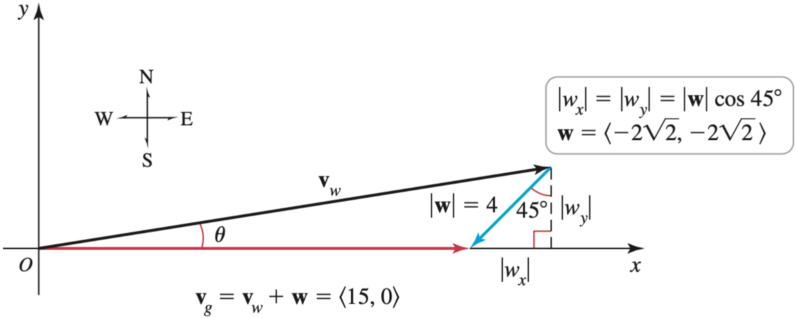
- Water in river moves at 4 miles/h SW
- Boat moves 15 miles/h E

Question:
Find the speed of the boat and its heading.

Notation: We set

- v_g = velocity wrt the shore
- w = vector representing the current
- v = velocity of the boat

Speed of boat in current (2)



Speed of boat in current (3)

Computations: We have

- $v_g = \langle 15, 0 \rangle$
- $w = \langle 4\cos(225), \, -4\sin(225) \rangle = \left\langle -2\sqrt{2}, -2\sqrt{2} \right\rangle$
- $v_g = v + w$

Conclusion: We get

$$v = \left\langle 15 + 2\sqrt{2}, \, 2\sqrt{2} \right\rangle .$$

Thus

$$|v| \simeq 18, \quad \theta = \tan^{-1}\left(\frac{2\sqrt{2}}{15 + 2\sqrt{2}} \right) \simeq 9^{\circ}$$

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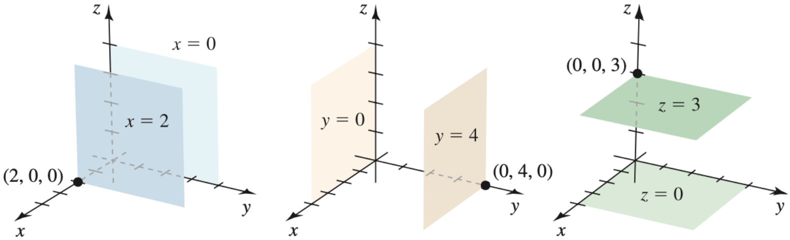
Planes

Basic rule:
Most shapes in $\mathbb{R}^3$ are similar to their $\mathbb{R}^2$ counterparts

Example of shape in $\mathbb{R}^2$:
Equation $x = 2$, which gives a line

Example of shape in $\mathbb{R}^3$:
Equation $x = 2$, which gives a plane

Geometric representation of planes



Circles and spheres

Circle: In $\mathbb{R}^2$, the equation

$$(x-a)^2 + (y-b)^2 = r^2$$

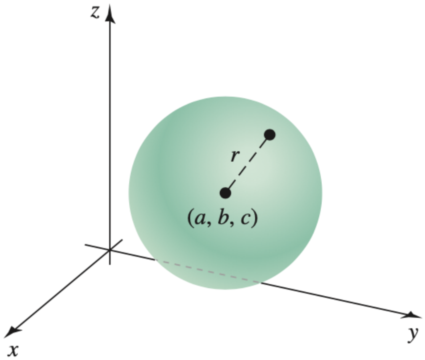
corresponds to a circle with center (a, b) and radius r

Sphere: In $\mathbb{R}^3$, the equation

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

corresponds to a sphere with center (a, b, c) and radius r

Sphere: illustration



Sphere: $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

Ball: $(x-a)^2+(y-b)^2+(z-c)^2\leq r^2$

Examples of sphere (1)

Standard form: The equation

$$(x-7)^2 + (y+6)^2 + z^2 = 10$$

represents a sphere with center $(7, -6, 0)$ and radius $\sqrt{10}$.

Non standard form: The equation

$$x^2 + y^2 + z^2 - 14x + 12y + 25 = 0$$

represents a sphere with center $(7, -6, 0)$ and radius $\sqrt{60}$.

Proof:
Complete the squares

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Definition of dot product

Definition 3.

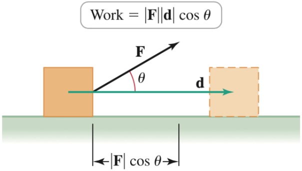
Let

- $\mathbf{u}, \mathbf{v}$ vectors in $\mathbb{R}^3$
- $\theta \in [0, \pi]$ angle between $\mathbf{u}$ and $\mathbf{v}$

Then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}|\,\cos(\theta)$$

Motivation: Work of a force



Analytic expression for the dot product

Theorem 4.

Let

- $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$
- $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$

Then we have

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

Example of dot product

Computation of dot product: If

$$\mathbf{u} = \langle 1, 2, 3 \rangle \,, \quad \mathbf{v} = \langle 4, 5, 6 \rangle \,,$$

then according to Theorem 4,

$$\mathbf{u} \cdot \mathbf{v} = 32$$

Angle between **u** and **v**: According to Definition 7

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| \, |\mathbf{v}|} = \frac{32}{\sqrt{14 \times 77}}.$$

Thus

$$\theta \simeq 13^{\circ}$$

Orthogonal vectors

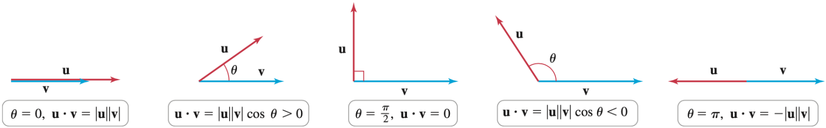
Definition 5.

Let

- $\mathbf{u}, \mathbf{v}$ vectors in $\mathbb{R}^3$

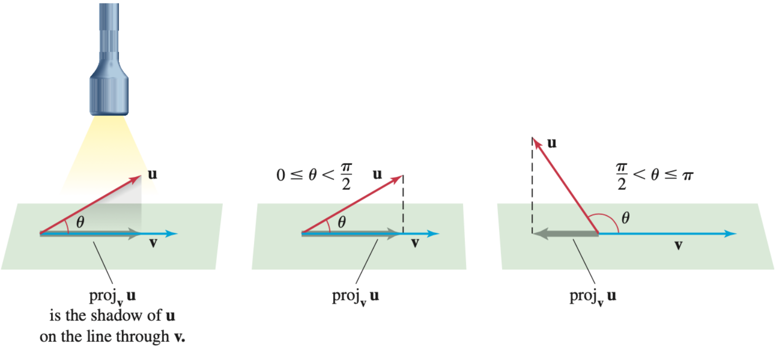
Then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if

$$\mathbf{u} \cdot \mathbf{v} = 0$$



Orthogonal projection (1)

Question answered by projecting:
How much of $\mathbf{u}$ points into the direction of $\mathbf{v}$?



Orthogonal projection (2)

Definition 6.

Let

- $\mathbf{u}, \mathbf{v}$ vectors in $\mathbb{R}^3$
- $\theta \equiv$ angle between $\mathbf{u}$ and $\mathbf{v}$

Then the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$ is

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \text{scal}_{\mathbf{v}}(\mathbf{u}) \, \frac{\mathbf{v}}{|\mathbf{v}|},$$

where

$$\text{scal}_{\mathbf{v}}(\mathbf{u}) = |\mathbf{u}| \, \cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|}$$

Orthogonal projection (3)

Remark on the projection formula:

- $\text{scal}_{\mathbf{v}}(\mathbf{u})$ is the signed magnitude of $\text{proj}_{\mathbf{v}}(\mathbf{u})$
- $\frac{\mathbf{v}}{|\mathbf{v}|}$ is the direction given by $\mathbf{v}$

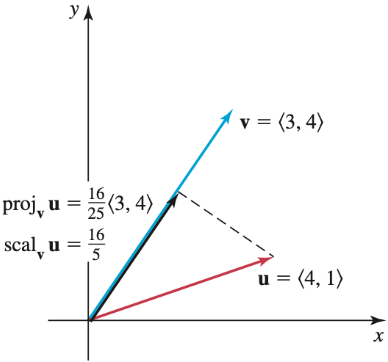
Another expression for the projection:

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \right) \mathbf{v},$$

Orthogonal projection (4)

Example of projection: Consider

$$\mathbf{u} = \langle 4, 1 \rangle, \qquad \mathbf{v} = \langle 3, 4 \rangle$$



Orthogonal projection (5)

Computation through definition: We have

$$\text{scal}_{\mathbf{v}}(\mathbf{u})\,\frac{\mathbf{v}}{|\mathbf{v}|} = \frac{16}{5}, \quad \frac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

Hence

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \frac{16}{25}\,\langle 3, 4 \rangle\,.$$

Computation through other expression: We have

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v} = \frac{16}{25}\,\langle 3, 4 \rangle$$

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Definition of cross product

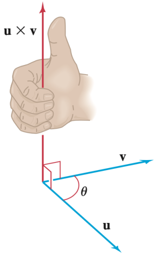
Definition 7.

Let

- $\mathbf{u}, \mathbf{v}$ vectors in $\mathbb{R}^3$, with angle $\theta \in [0, \pi]$

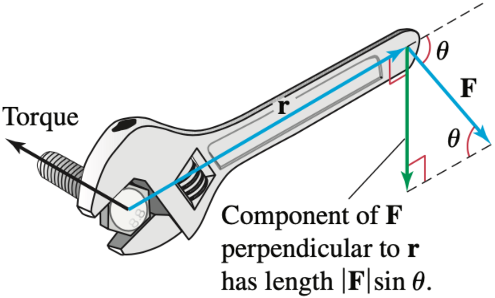
Then $\mathbf{u} \times \mathbf{v}$ is a vector such that

1. Magnitude is $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\,\sin(\theta)$.
2. Direction: given by right hand rule.



Cross product: illustration

Motivation: Torque



Formula for cross product

Formula: We have

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Example of cross product: If

$$\mathbf{u} = \langle 2, 1, 1 \rangle, \qquad \mathbf{v} = \langle 5, 0, 1 \rangle,$$

then

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & 1 & 1 \\ 5 & 0 & 1 \end{vmatrix} = \langle 1, 3, -5 \rangle$$

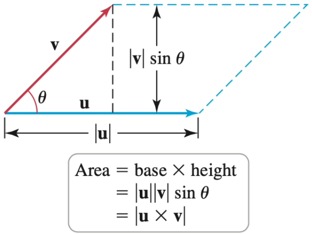
Properties of the cross product

Antisymmetry: We have

$$\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v})$$

Areas: We also have

$$|\mathbf{u} \times \mathbf{v}| = \text{Area of parallelogram with } \mathbf{u}, \mathbf{v} \text{ as intersecting sides}$$



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Parametric form of the equation of a line

Proposition 8.

Let

- $P_0 = (x_0, y_0, z_0)$ point in $\mathbb{R}^3$
- $\mathbf{v} = \langle a, b, c \rangle$ vector

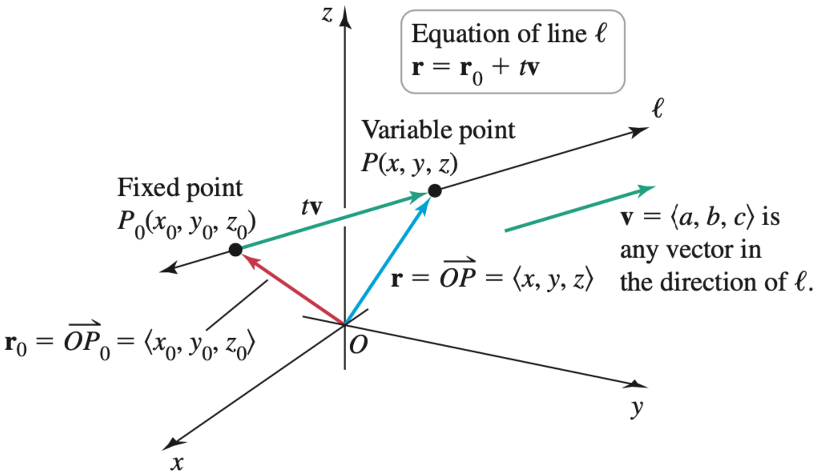
Then the parametric equation of a line passing through P_0 in the direction of $\mathbf{v}$ is

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle, \qquad t \in \mathbb{R}.$$

For coordinates, we get

$$\begin{cases} x &= x_0 + a\,t \\ y &= y_0 + b\,t \\ z &= z_0 + c\,t \end{cases}$$

Line in space: illustration



Example of parametric form (1)

Problem: Find the equation of a line

- Through point $(1, 2, 3)$
- Along $\mathbf{v} = \langle 4, 5, 6 \rangle$

Example of parametric form (2)

Vector form:

$$\langle x, y, z \rangle = \langle 1, 2, 3 \rangle + t \langle 4, 5, 6 \rangle , \qquad t \in \mathbb{R}.$$

Coordinates form:

$$\begin{cases} x &= 1 + 4\,t \\ y &= 2 + 5\,t \\ z &= 3 + 6\,t \end{cases}$$

Example of line segment (1)

Problem: Find the equation of line segment

From $P(0, 1, 2)$ to $Q(-3, 4, 7)$

Example of line segment (2)

Direction vector: $\mathbf{v} = \vec{PQ} = \langle -3, 3, 5 \rangle$

Initial vector: $\vec{OP} = \langle 0, 1, 2 \rangle$

Equation:

$$\langle x, y, z \rangle = \langle 0, 1, 2 \rangle + t \langle -3, 3, 5 \rangle , \qquad t \in [0, 1].$$

Points of intersection for lines

Problem:
Determine if ℓ_1 and ℓ_2 intersect and find point of intersection, with

$$\begin{aligned}\ell_1 &: \quad x = 2 + 3t, \quad y = 3t, \quad z = 1 - t \\ \ell_2 &: \quad x = 4 + 2s, \quad y = -3 + 3s, \quad z = -2s\end{aligned}$$

Points of intersection for lines (2)

Step 1: Check that $\mathbf{v}_1$ not parallel to $\mathbf{v}_2$. Here

$$\mathbf{v}_1 = \langle 3, 3, -1 \rangle \,, \quad \text{not parallel to} \quad \mathbf{v}_2 = \langle 2, 3, -2 \rangle$$

Step 2: Equation for intersection

$$\begin{cases} 2 + 3t & = 4 + 2s \\ 3t & = -3 + 3s \\ 1 - t & = -2s \end{cases}$$

This system has no solution

$\hookrightarrow$ ℓ_1 does not intersect ℓ_2

Points of intersection for lines (3)

Some conclusions:

1. If $\mathbf{v}_1 \parallel \mathbf{v}_2$,
   $\hookrightarrow$ ℓ_1 does not intersect ℓ_2
2. Even if $\mathbf{v}_1$ not parallel to $\mathbf{v}_2$,
   $\hookrightarrow$ we can have that ℓ_1 does not intersect ℓ_2
3. In the latter case, we say that the lines ℓ_1 and ℓ_2 are skewed

Equation of a plane in $\mathbb{R}^3$

Proposition 9.

Let

- $P_0 = (x_0, y_0, z_0)$ point in $\mathbb{R}^3$
- $\mathbf{n} = \langle a, b, c\rangle$ vector

Then the parametric equation of a plane passing through P_0 with normal vector $\mathbf{n}$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Remarks on plane equations

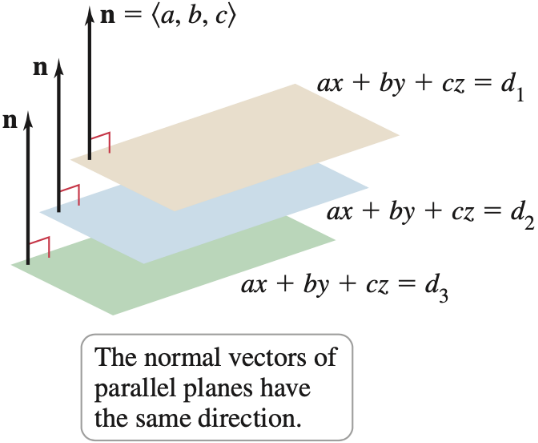
Plane and dot product: The plane is the set of points P such that

$$\vec{P_0P} \cdot \mathbf{n} = 0$$

Other expression for the plane equation:

$$a\,x + b\,y + c\,z = d, \quad \text{with} \quad d = a\,x_0 + b\,y_0 + c\,z_0$$

Plane: illustration



Computing plane equations (1)

Problem: Compute the equation of the plane containing

$$\mathbf{u} = \langle 0, 1, 2 \rangle, \quad \mathbf{v} = \langle -1, 3, 0 \rangle, \quad P_0(-4, 7, 5)$$

Computing plane equations (2)

Computing the normal vector:

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = -\langle 6, 2, -1 \rangle$$

Equation for the plane:

$$6x + 2y - z = -15$$

Intersecting planes (1)

Problem: Find an equation of the line of intersection of the planes

$$Q : x + 2y + z = 5$$

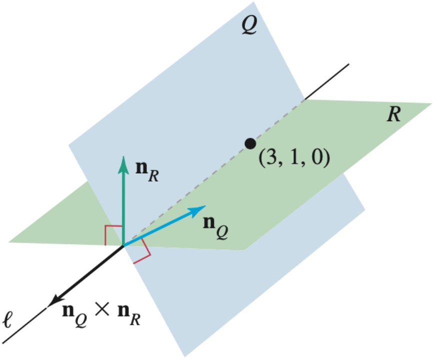
and

$$R : 2x + y - z = 7$$

Strategy:

1. Find a point P_0 in $Q \cap R$
   $\hookrightarrow$ Solve system
2. Find the direction $\mathbf{v}$ of $Q \cap R$
   $\hookrightarrow$ Given by $\mathbf{v} = \mathbf{n}_Q \times \mathbf{n}_R$

Intersecting planes (2)



$\mathbf{n}_Q \times \mathbf{n}_R$ is a vector perpendicular to $\mathbf{n}_Q$ and $\mathbf{n}_R$.
Line ℓ is perpendicular to $\mathbf{n}_Q$ and $\mathbf{n}_R$.
Therefore, ℓ and $\mathbf{n}_Q \times \mathbf{n}_R$ are parallel to each other.

Intersecting planes (3)

System to find P_0 Take (e.g) $z = 0$. Then we get

$$x + 2y = 5, \qquad 2x + y = 7$$

Intersection: We find

$$P_0(3, 1, 0)$$

Intersecting planes (4)

Direction of the line: We have

$$\mathbf{n}_Q \times \mathbf{n}_R = \langle -3, 3, -3 \rangle$$

Thus we can take

$$\mathbf{v} = \langle 1, -1, 1 \rangle$$

Equation of the line:

$$\langle x, y, z \rangle = \langle 3 + t, 1 - t, t \rangle , \qquad t \in \mathbb{R}.$$

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Cylinder

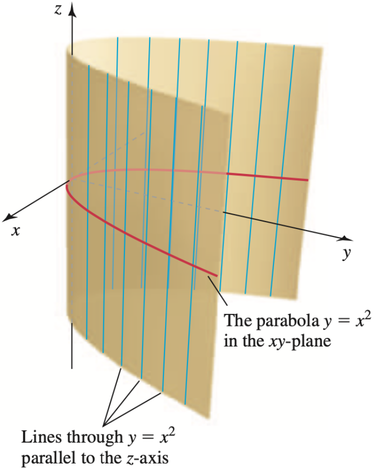
Shapes in $\mathbb{R}^3$:
Surfaces S whose equation contain the 3 variables x, y, z

Free variable: If a variable is missing from the equation of S
$\hookrightarrow$ It can take any value in $\mathbb{R}$ and is called free

Cylinder: Surface S with a free variable

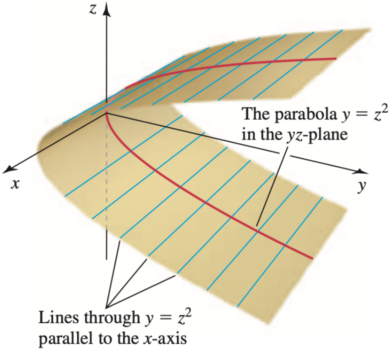
Example 1 of cylinder

Equation: $y = x^2$



Example 2 of cylinder

Equation: $y = z^2$



Trace

Definition 10.

Let

- S a surface in $\mathbb{R}^3$

Then

1. A trace of S is the set of points at which S intersects a plane that is parallel to one of the coordinate planes.
2. The traces in the coordinate planes are called the xy-trace, the xz-trace, and the yz-trace

Elliptic paraboloid (1)

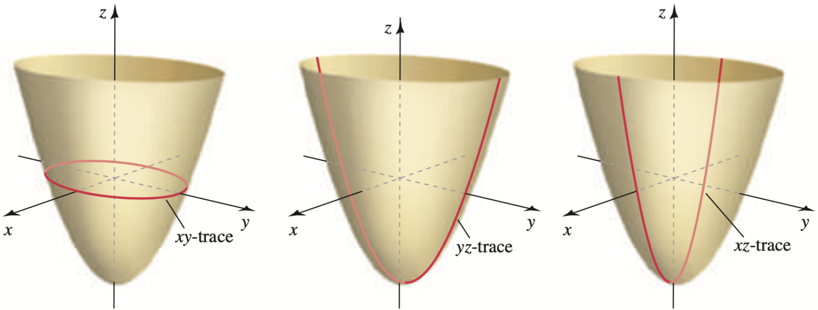
Problem: Graph the surface

$$z = \frac{x^2}{16} + \frac{y^2}{4}$$

Traces:

- xy-trace: ellipse, whenever $z_0 \geq 0$
- xz-trace: parabola
- yz-trace: parabola

Elliptic paraboloid (2)



Graphing a cylinder (1)

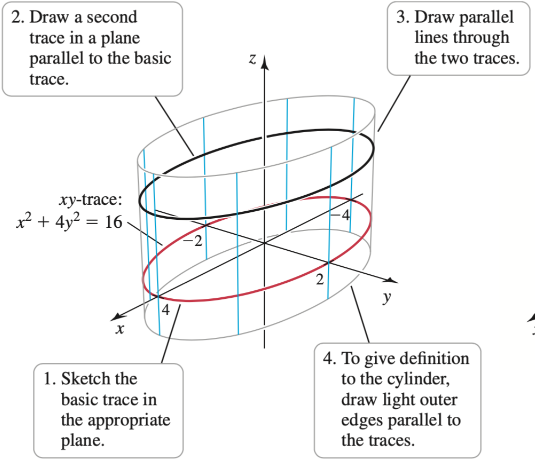
Problem: Graph the cylinder

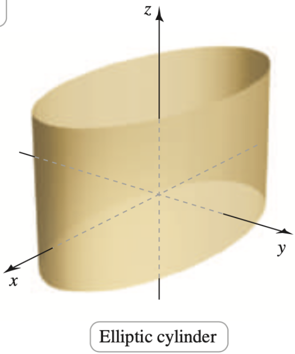
$$S: \; x^2 + 4y^2 = 16$$

Graphing a cylinder (2)

1. Cylinder feature: Since z absent from equation
   $\hookrightarrow$ S is a cylinder with lines $\parallel$ to z axis
2. xy-trace: Ellipse of the form
$$\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1$$
3. Draw:
   - 1 trace in xy-plane
   - Another trace in e.g plane $z = 1$
   - Lines between those 2 traces

Graphing a cylinder (3)





Quadric surfaces

Analytic definition: Given by an equation of the form

$$S: \; Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

Strategy for graphing:

1. Intercepts. Determine the points, if any, where the surface intersects the coordinate axes.
2. Traces. Finding traces of the surface helps visualize the surface.
3. Completing the figure. Draw smooth curves that pass through the traces to fill out the surface.

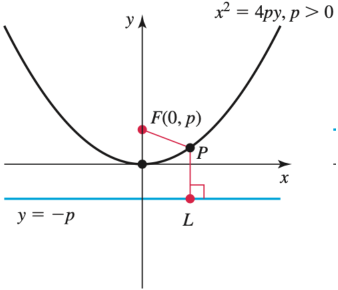
2d conic sections: parabola

Prototype of standard equation:

$$y = \frac{x^2}{4p}$$

Geometric definition:

$$\{P;\ \text{dist}(P, F) = \text{dist}(P, L)\}$$



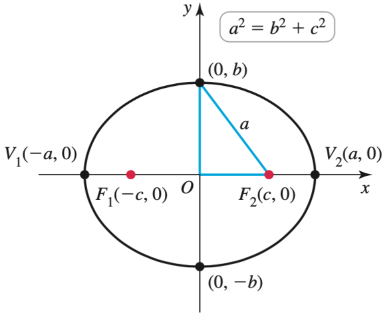
2d conic sections: ellipse

Prototype of standard equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Geometric definition: With $a^2 = b^2 + c^2$,

$$\{P;\ \text{dist}(P, F_1) + \text{dist}(P, F_2) = 2a\}$$



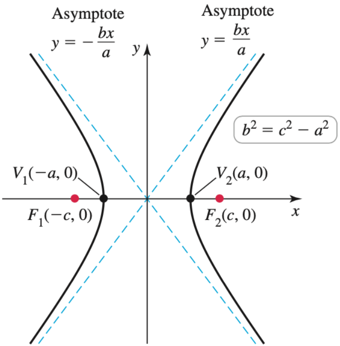
2d conic sections: hyperbola

Prototype of standard equation:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Geometric definition: With $a^2 = c^2 - b^2$,

$$\{P;\ \text{dist}(P, F_2) - \text{dist}(P, F_1) = \pm 2a\}$$



Hyperboloid of one sheet (1)

Equation:

$$\frac{x^2}{4} + \frac{y^2}{9} - z^2 = 1$$

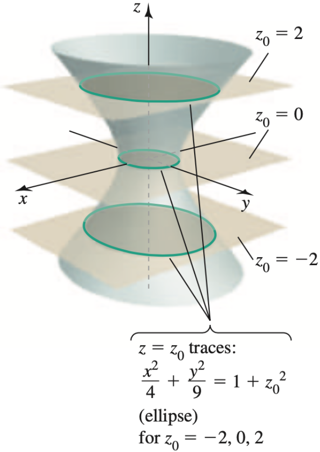
Intercepts:

$$(0, \pm 3, 0), \qquad (\pm 2, 0, 0)$$

Hyperboloid of one sheet (2)

Traces in xy-planes: Ellipses of the form

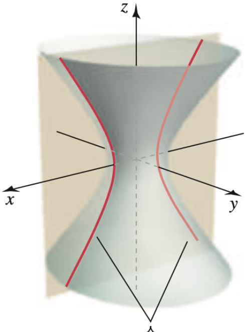
$$\frac{x^2}{4} + \frac{y^2}{9} = 1 + z^2$$



Hyperboloid of one sheet (3)

Traces in xz-planes: For $y = 0$, hyperbola

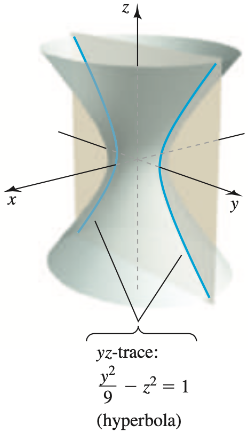
$$\frac{x^2}{4} - z^2 = 1$$



Hyperboloid of one sheet (4)

Traces in *yz*-planes: For $x = 0$, hyperbola

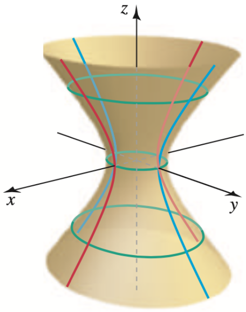
$$\frac{y^2}{9} - z^2 = 1$$



Hyperboloid of one sheet (5)

Equation:

$$\frac{x^2}{4} + \frac{y^2}{9} - z^2 = 1$$



Hyperbolic paraboloid (1)

Equation:

$$z = x^2 - \frac{y^2}{4}$$

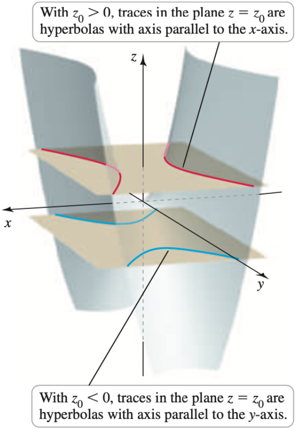
Intercept:

$$(0, 0, 0)$$

Hyperbolic paraboloid (2)

Traces in xy-planes: Hyperbolas (axis according to $z > 0$, $z < 0$) of the form

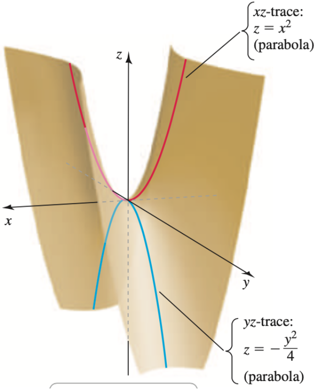
$$x^2 - \frac{y^2}{4} = z_0$$



Hyperbolic paraboloid (3)

Traces in xz-planes: For $y = y_0$, upward parabola

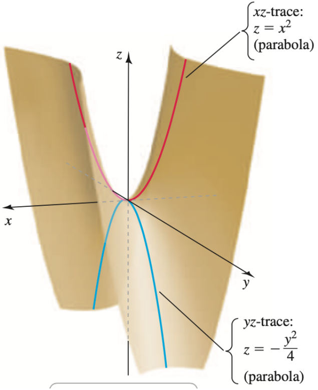
$$z = x^2 - \frac{y_0^2}{4}$$



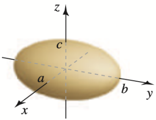
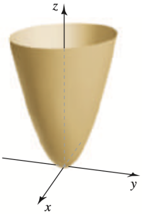
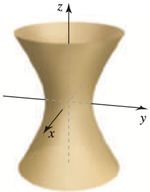
Hyperbolic paraboloid (4)

Traces in yz-planes: For $x = x_0$, downward parabola

$$z = -\frac{y^2}{4} + x_0^2$$



Summary of quadric surfaces (1)

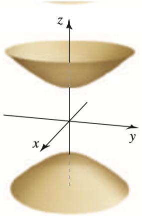
Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	All traces are ellipses.	
Elliptic paraboloid	$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ are ellipses for all z_0. Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	

Summary of quadric surfaces (2)

Hyperboloid of two sheets

$$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

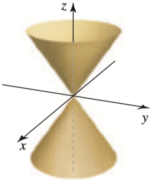
Traces with $z = z_0$ with $|z_0| > |c|$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.



Elliptic cone

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$$

Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.



Hyperbolic paraboloid

$$z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$$

Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.