$\boxed{\text{FALL 18 FINAL}}$

Pb 21 $\quad f(x) = e^x + x^2$

$A(D)$ for x^2 : D^3

$A(D)$ for e^x : $D-1$

$A(D)$ for f :

$(D-1)\,D^3$

Answer: E

Pb 22

(i) $f_1 = \begin{pmatrix} 1 \\ |t| \end{pmatrix}$ $f_2 = \begin{pmatrix} |t| \\ t^2 \end{pmatrix}$

Then $f_2 = |t| f_1$, and we don't have $f_2 = c f_1$, in spite of the fact that $W[f_1, f_2] = 0 \rightsquigarrow (f_1, f_2)$ lin. independent

(ii) $x_1 = \begin{pmatrix} e^t \\ -2e^t \end{pmatrix}$ $x_2 = \begin{pmatrix} -2e^t \\ 4e^t \end{pmatrix}$

Then $x_2 = -2x_1$, thus J_2 lin. dep.

(iii) $x_1 = \begin{pmatrix} e^t \\ 2e^t \end{pmatrix}$ $x_2 = \begin{pmatrix} 2e^{2t} \\ e^{2t} \end{pmatrix}$

Then

$$W[x_1, x_2](t) = -3e^{3t} \neq 0$$

Thus J_3 lin. indep

Answer: C

Pb 23 $\quad x'(t) = \overbrace{\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}}^{A} x(t) \qquad x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Eigenvalue decomposition for A:

$\lambda_1 = 1, \qquad v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\lambda_2 = 2 \qquad v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Then the general solution is

$$x(t) = c_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

With initial condition we get

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Thus $\quad c_1 = -1, \quad c_2 = 1$ and

$$x(t) = -e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

and $\quad x_2(t) = e^{2t}$

Answer: $\quad$ B

Pb 24 We are given

$$X(t) = \begin{pmatrix} 2e^{t} & e^{2t} \\ e^{t} & e^{2t} \end{pmatrix} \qquad f(t) = \begin{pmatrix} e^{t} \\ -e^{t} \end{pmatrix}$$

Then $v(t)$ is such that $Xv' = f$.

We wish to find $v_1(t)$. We have

$$\det(X(t)) = e^{3t}$$

Thus

$$v_1'(t) = e^{-3t} \begin{vmatrix} e^{t} & e^{2t} \\ -e^{t} & e^{2t} \end{vmatrix} = 2$$

We get

$$v_1(t) = 2t$$

Answer: A