MA262 — PRACTICE PROBLEMS FOR EXAM I — SPRING 2022
BASED ON OLD MA262 EXAMS WHICH CAN BE FOUND AT

https://www.math.purdue.edu/academic/courses/oldexams.php?course=MA26200

This is a collection of problems from old exams, the textbook and some other exercises. We have been using a new textbook book since the fall 2020 and some old exam problems do not match the notation or the content.

1. Let $y(x)$ satisfy

$$y' + \frac{4}{x}y = 12x,$$
$$y(1) = 8.$$

Then $y(2)$ is equal to

A. $\frac{25}{4}$

B. $\frac{32}{9}$

C. $\frac{67}{8}$

D. $\frac{23}{2}$

E. $\frac{47}{8}$

2. Let $y(x)$ be the solution of

$$(y^2 + 2x + \cos x)dx + (2xy + \sin y)dy = 0,$$
$$y(0) = \pi.$$

If we denote $y(\frac{\pi}{2}) = Y$, we can say that Y satisfies the following equation:

A. $\frac{\pi}{2}Y^2 + \frac{\pi^2}{4} - \cos Y = 0.$

B. $\frac{\pi}{2}Y^3 + \frac{\pi^2}{4} - \cos Y = 1.$

C. $\frac{\pi}{2}Y^2 + \frac{\pi^2}{4} + \sin Y = 0.$

D. $\frac{\pi}{2}Y^2 - \frac{\pi^2}{4} + \sin Y = 0.$

E. $\frac{\pi}{4}Y^2 + \frac{\pi^2}{4} - (\cos Y)^2 = 0.$

3. Let $y(x)$ satisfy

$$\begin{aligned} &y'(x) + \frac{2x}{1+x^2} y = -2xy^2, \\ &y(0) = 1. \end{aligned}$$

Then $y(1)$ is equal to

A. $y(1) = \dfrac{1}{2 + 2\ln 2}$

B. $y(1) = \dfrac{1}{1 + \ln 2}$

C. $y(1) = 2(\ln 2) + 1$

D. $y(1) = 2 + (\ln 2)$

E. $y(1) = \dfrac{1}{3 + \ln 2}$

4. The solution of the differential equation

$$\frac{dy}{dx} = \frac{x^3 + 4y^3}{3xy^2}, \quad x > 0$$

is defined implicitly by the following equation:

A. $x^3 + y^3 - Cxy^3 = 0$

B. $(x+y)^3 - Cx = 0$

C. $x^3 + y^3 - Cx^4 = 0$

D. $x + y - Cx^2 = 0$

E. $1 + y^3 - Cx = 0$

5. Let $y(x)$ satisfy the following second order differential equation

$$yy'' = 3(y')^2,$$
$$y(0) = 1, \quad y'(0) = 1.$$

Then $y(\frac{3}{8})$ is equal to

A. $y(\dfrac{3}{8}) = 1$

B. $y(\dfrac{3}{8}) = 2$

C. $y(\dfrac{3}{8}) = 3$

D. $y(\dfrac{3}{8}) = 4$

E. $y(\dfrac{3}{8}) = 5$

6. Find the general solution of the equation $y'' + 64y = 0$.

A. $y(x) = \cos(4x) + C$

B. $y(x) = \cos(4x) + \sin x$

C. $y(x) = A\cos(4x) + B\sin(4x)$

D. $y(x) = A\cos(4x) + B\sin(5x)$

E. $y(x) = A\cos(3x) + B\sin(4x) + C\sin(2x)$

7. Find the solution of the initial value problem

$$xy' = y + \frac{5}{4}(x^4 y)^{\frac{1}{5}},$$
$$y(1) = 1.$$

A. $y(x) = x(1 + \ln x)^{\frac{5}{4}}$

B. $y(x) = x^2(1 + \ln x)^{\frac{3}{4}}$

C. $y(x) = x(1 + x\ln x)^{\frac{1}{4}}$

D. $y(x) = x(1 + 3\ln x)^{\frac{5}{4}}$

E. $y(x) = x(1 + 5\ln x)^{\frac{5}{4}}$

8. An object with initial temperature 32F is placed in a refrigerator whose temperature is a constant 0F. An hour later the temperature of the object is 16F. What will its temperature be four hours after it is placed in the refrigerator? Hint: Newton's law of cooling $\frac{dT}{dt} = -k(T - T_m)$.

A. 1

B. 2

C. 3

D. 4

E. 5

9. The population of a certain species obeys the equation

$$\frac{dp}{dt} = 10(200 - p)(p - 300).$$

If the initial population is 100, what will the approximate value of the population be after a long time?

A. 100

B. 200

C. 300

D. 150

E. The population will be extinct .

10. A tank originally contains 100 gal of water with a salt concentration of $\frac{1}{2}$ lb/gal. A solution containing a salt concentration of 2 lb/gal enters at a rate of 2 gal/min and the well-stirred mixture is pumped out at the rate of 1 gal/min. The amount of salt in the tank after 50 min is

A. 0 lb

B. $400 - 350e^{\frac{1}{2}}$ lb

C. e^{-2} lb

D. 100 lb

E. 200 lb

11. Consider the system

$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 3x_2 + 2x_3 = 5$$
$$2x_1 + 3x_2 + (k^2 - 2)x_3 = k + 7$$

Determine all the values of the constant k for which the above system has no solutions.

A. $k = -2$

B. $k = 2$

C. $k \neq -2$

D. $k \neq 2$

E. $k = 3$

17. Solve the differential equation

$$(2xy + x^3)dx + (x^2 + y^3 + 2)dy = 0, \quad y(0) = 2.$$

A. $x^2y + 2y = 4$

B. $x^4 + 2y = 8$

C. $x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 + 2y = 8$

D. $x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 = 0$

E. $\frac{1}{4}x^4 + \frac{1}{4}y^4 = 8$

1. If $xy' - 3y = x^3$ and $y(1) = 1$, then $y(e) =$

A. e^4
B. e^{-2}
C. $2e^{-3}$
D. $2e^3$
E. e^3

2. The general solution of

$$(2x^2y)y' = -3x^2 - 2xy^2$$

is:

A. $x^2y^3 + y^3 = C$
B. $x^2y^2 + x^3 = C$
C. $x^2y^2 = C$
D. $x^3y^2 + x^2 = C$
E. $x^2y^3 + x = C$

2. If the system

$$\begin{aligned} 3x + y - 5z &= a \\ 2x + 2y - 3z &= b \\ x - y - 2z &= c \end{aligned}$$

is consistent, what can we conclude about a, b and c?

A. $c^2 = a^2$

B. $a + b = 6$

C. $c = 3$

D. $c = a - b$

E. $c = a + b$

3. The general solution of $xy' - y = x^2e^x$ is

A. $y = xe^x + cx$

B. $y = x^2e^x - xe^x + cx$

C. $y = xe^x - cx^2$

D. $y = x^2e^x + xe^x + cx$

E. None of the above

4. The solution of $(3x^2 + y)dx + (x + 2y)dy = 0$ passing through the point $(1, 1)$ is

A. $x^2 + xy + y^2 = 3$

B. $x^2 + xy + y^3 = 3$

C. $x^2 + x + y^2 = 3$

D. $x^3 + xy + y^2 = 3$

E. $x^3 + x^2y + y^3 = 3$

2. If y is the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x(y-2)}{x^2+1}, \qquad y(0) = 4,$$

then $y(1) =$

A. 4
B. 6
C. 8
D. 10
E. 12

2. (8 points) For $x > 0$, the solution to the equation

$$\left(\frac{2y}{x} + 2x\right) + (2\ln(x) - 3)\, y' = 0$$

is given implicitly by

A. $y(2\ \ln(x) - 3) + x^2 = c$

B. $y(2\ \ln(x) + 3) - 3x^2 = c$

C. $\ln(x) - x^3 - 3y + c = 0$

D. $y(\frac{2}{x} + \frac{3}{2}) + x^2 + c = 0$

E. $\frac{1}{x} + x^2 - y^2 + c = 0$

10. For which value of c does the following system have infinitely many solutions?

$$\begin{cases} 3x - 2y + 5z = 1 \\ 2y + z = 1 \\ -3x + 6y + cz = 1 \end{cases}$$

A. -3

B. -1

C. $1/2$

D. $5/2$

E. 2