MA 262 - DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

REVIEW PROBLEMS - FINAL

Problem 1. Find the general solution of the following equation:

$$(y\cos(x) + 2x\,e^{y}) + \big(\sin(x) + x^2 e^{y} - 1\big)\,y' = 0$$

Problem 2. We consider the following equation:

$$x^3 y' + 4x^2 y = e^{-x}.$$

Find the general form of the solution.

Problem 3. Solve the initial value problem

$$y' = \frac{2\cos(x)}{3+2y}, \qquad y(0) = 1$$

Problem 4. Transform the following equation into a linear equation by substitution.

$$t^2 y' + 2t\,y - y^3 = 0.$$

Problem 5. A tank initially contains 10 L of pure water in which 5g of salt is dissolved. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t.

Problem 6. Using a substitution, transform the following equation into a separable equation.

$$y' = \frac{2x + (x^3 + y^3)^{1/3}}{y}.$$

Problem 7. Give the general solution of the following equation:

$$y'' - \frac{2}{x} y' = 18x^4.$$

Problem 8. Determine the number of solutions of the following system according to the values of $k \in \mathbb{R}$:

$$\begin{array}{rrrl} x_1 & +2x_2 & -x_3 & = 3 \\ 2x_1 & +5x_2 & +x_3 & = 7 \\ x_1 & +x_2 & -k^4 x_3 & = k^2 \end{array}$$

Problem 9. Let A and B be two 3×3 matrices defined by

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 4 & -2 \\ -3 & 5 & 7 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ 3 & -5 & 2 \end{bmatrix}$$

Compute $\det(B^2 A^{-1})$.

Problem 10. Let A be the matrix defined by

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 1 & 1 \\ 3 & -1 & 0 \end{bmatrix}$$

Compute the value of the $(3, 1)$-element of A^{-1}.

Problem 11. Describe colspace(A) for A given as

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 11 & 21 \\ 3 & 7 & 13 \end{bmatrix}$$

Problem 12. Determine the dimension of the space spanned by the following vectors.

$$\begin{bmatrix} 1 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 0 \\ 2 \\ -2 \end{bmatrix}$$

Problem 13. Let p_1 and p_2 be the polynomials defined by

$$p_1(x) = 1 - ax, \qquad p_2(x) = 1 + x$$

Determine the values of a such that p_1 and p_2 are linearly independent.

Problem 14. For solutions of differential equations or systems in $\mathbb{R}^n$ or $M_{n,n}(\mathbb{R})$, establish a criterion to know if the solution set is a vector space or not.

Problem 15. Let T be a linear transformation such that

$$T(\mathbf{v}_1 + \mathbf{v}_2) = 3\mathbf{v}_1 - \mathbf{v}_2, \qquad T(2\mathbf{v}_1 + \mathbf{v}_2) = \mathbf{v}_1 + 2\mathbf{v}_2.$$

Find the expression for $T(a\mathbf{v}_1 + b\mathbf{v}_2)$ for arbitrary a, b.

Problem 16. Determine if the following matrix is defective:

$$A = \begin{bmatrix} 6 & 3 & -4 \\ -5 & -2 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

Problem 17. Compute $\dim(\ker(T)) + 2\dim(\mathrm{Rng}(T))$ for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$A = \begin{bmatrix} 1 & -2 & 5 \\ 2 & -3 & -1 \\ 5 & -8 & -1 \end{bmatrix}$$

Problem 18. Solve the following initial value problem:

$$y'' + 4y = 0, \qquad y(0) = 0, \quad y'(0) = 1.$$

Find $y(\frac{\pi}{4})$.

Problem 19. Given that $y_1(x) = x^{-1}$ is a solution of

$$2x^2y'' + 3xy' - y = 0, \qquad x > 0,$$

find a fundamental set of solutions.

Problem 20. Find the general solution of the following equation:

$$y'' + y = \tan(t)$$

Problem 21. Find the general form of a particular solution for the following equation:

$$y^{(4)} - y = 3t + \cos(t)$$

Problem 22. Solve the following initial value problem:

$$\mathbf{x}' = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Problem 23. Find a particular solution $\mathbf{x}_p$ of the following system:

$$\mathbf{x}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix}$$