Rmk on ct rv:

Discrete	Continuous
pmf p	density f
$\sum p(x_i)$	$\int f(x)\,dx$
$P(a \le X \le b) = \sum_{x_i=a}^{b} p(x_i)$	$P(a \le X \le b) = \int_a^b f(x)\,dx$

Justification: $P(X=a) = 0$

$$P(X=a) = \int_a^a f(x)\,dx = 0$$

$P(a \le X \le a)$

Rmk on radio tube

For any density function f, we should have

$$\int_{\mathbb{R}} f(x)\,dx = \int_{-\infty}^{\infty} f(x)\,dx = 1$$

(equivalent of $\sum_{x_i \in E} p(x_i) = 1$)

Here

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{100}{x^2} \mathbb{1}_{(100,\infty)}(x)\,dx$$

$$= \int_{100}^{\infty} \frac{100}{x^2}\,dx$$

$$= -\frac{100}{x}\Big|_{100}^{\infty}$$

$$= -0 + \frac{100}{100} = 1$$

Radio tube

Definitions. We let

$X_i =$ lifetime of tube $\# i$

$E_i =$ "tube $\# i$ has to be replaced within 1st 150h of operation"

Then

$$\mathbb{P}(E_i) = \mathbb{P}(X_i \leq 150)$$

$$= F_{X_i}(150)$$

$$= \int_{-\infty}^{150} f(x)\, dx$$

$$= \int_{\infty}^{150} \frac{100}{x^2} \mathbb{1}_{(100,\infty)}(x)\, dx$$

$$= 100 \int_{100}^{150} \frac{dx}{x^2} = -100 \left. \frac{1}{x} \right|_{100}^{150}$$

$$= 100 \left[\frac{1}{100} - \frac{1}{150} \right] = 1 - \frac{2}{3} = \frac{1}{3}$$

Radio tube (ctd)

Let Z = # tubes which have to be replaced within 1st 150h

We wish to compute

$$\mathbb{P}(Z = 2)$$

Q: What is the distribution of Z?

A: We have

$$Z = \sum_{i=1}^{5} \underbrace{\mathbb{1}_{E_i}}_{\mathcal{B}(\frac{1}{3})} \quad (\text{\# successes})$$

Thus $Z \sim Bin(5, \frac{1}{3})$ and

$$P(Z=2) = \binom{5}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3$$

$$\simeq 33\%$$