Rmk on $E[X]$

Discrete

$$E[X] = \sum_{i \geq 1} x_i \, p(x_i)$$

Ct

$$E[X] = \int_{\mathbb{R}} x \, f(x) \, dx$$

Simple example

$$f(x) = 2x \, \mathbb{1}_{[0,1]}(x)$$

Thus

$$\begin{aligned} E[X] &= \int_{\mathbb{R}} x \, f(x) \, dx \\ &= \int_{\mathbb{R}} x \times 2x \, \mathbb{1}_{[0,1]}(x) \, dx \\ &= \int_0^1 2x^2 \, dx \\ &= \frac{2}{3} x^3 \Big|_0^1 = \frac{2}{3} \end{aligned}$$

Rmk $X \in [0,1]$ since $f(x) = 0$ if $x \notin [0,1]$

We also have $E[X] \in [0,1]$.

Rmk on Prop 4

Discrete case

$E[g(X)]$

$=$

$\sum_{i \geq 1} g(x_i)\, p(x_i)$

Ct case

$E[g(X)]$

$=$

$\int_{\mathbb{R}} g(x)\, f(x)\, dx$

Simple example (ctd)

$E[X^3] = \int_{\mathbb{R}} x^3 f(x)\, dx$

$= \int_{\mathbb{R}} x^3\, 2x\, \mathbb{1}_{[0,1]}\, dx$

$= \int_{\mathbb{R}} 2x^4\, \mathbb{1}_{[0,1]}\, dx$

$= \int_0^1 2x^4\, dx$

$= \frac{2}{5} x^5 \Big|_0^1 = \frac{2}{5}$

Simple example - Var

$$Var(X) = E[X^2] - (\underbrace{E[X]}_{\frac{2}{3}})^2$$

$$E[X^2] = \int_{\mathbb{R}} x^2 f(x)\, dx$$

$$= \int_{\mathbb{R}} x^2 \times 2x \, \mathbb{1}_{[0,1]}(x)\, dx$$

$$= \int_0^1 2x^3 \, dx$$

$$= \frac{2}{4} x^4 \Big|_0^1 = \frac{1}{2}$$

Therefore

$$Var(X) = E[X^2] - (E[X])^2$$

$$= \frac{1}{2} - \left(\frac{2}{3}\right)^2$$

$$= \frac{1}{18}$$