Rmk. If we sum just 1 T_i:

$$\Gamma(1, \lambda) = \mathcal{E}(\lambda)$$

Tail, Weibull r.v.

For $x \geq \nu$, we have

$$\bar{F}(x) = 1 - F(x) = P(X > x)$$

$$= \exp\left(- \left(\frac{x - \nu}{\alpha}\right)^{\beta} \right)$$

ν → Bottom of the distribution

β → Decay of distribution

α → variation coeff

Recall

$$\lambda(x) = \frac{f(x)}{\bar{F}(x)} = \text{"death rate"}$$

Then

β large $\rightsquigarrow$ we become old very fast

$E[X]$, $X \sim \text{Cauchy}(\alpha = 0)$

$$E[X] = \int_{\infty}^{\infty} x\, f(x)\, dx$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \overbrace{\frac{x}{1+x^2}}^{\rightsquigarrow g(x)}\, dx$$

Then one might think

$g(-x) = -g(x)$, thus

$\int_{-\infty}^{\infty} g(x)\, dx = 0$ (see $N(0,1)$)

However, $g(x) \overset{x\to\infty}{\sim} \frac{x}{x^2} = \frac{1}{x}$

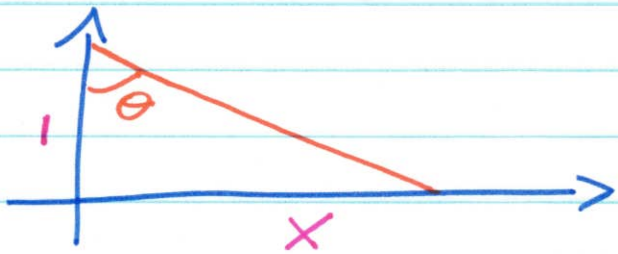
Since $\int_{\infty} \frac{1}{x}\, dx$ divergent,

we have $\int_{\infty} g(x)\, dx$ divergent

Thus

$E[X]$ not defined

Beam example



$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacente}} = \frac{x}{1} = x$$

We get $x = \tan(\theta)$

If we assume

$$\theta \sim U([-\tfrac{\pi}{2}, \tfrac{\pi}{2}])$$

then

$$X \sim \text{Cauchy}$$