Expected value of a Poisson r.v.

$$E[X(X-1)]$$

$$= \sum_{k=\cancel{0}\,2}^{\infty} k(k-1)\, e^{-\lambda} \frac{\lambda^k}{k!}$$

$$= \sum_{k=2}^{\infty} \frac{\lambda^k}{(k-2)!} e^{-\lambda} \qquad (\lambda^k = \lambda^{k-2} \times \lambda^2)$$

$$= e^{-\lambda} \sum_{k=2}^{\infty} \frac{\lambda^{k-2}}{(k-2)!} \lambda^2$$

cv: $j = k-2$

$$= \lambda^2 e^{-\lambda} \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} = e^{\lambda}$$

$$= \lambda^2 e^{-\lambda} e^{\lambda}$$

$$= \lambda^2$$

$$E[X(X-1)] = \lambda^2$$

Aim: compute $Var(X)$

$E[X^2] = ?$

$E[X(X-1)] = E[X^2 - X]$

linearity

$= E[X^2] - E[X]$

Thus

$E[X^2] = \overbrace{E[X(X-1)]}^{\lambda^2} + \overbrace{E[X]}^{\lambda}$

$E[X^2] = \lambda^2 + \lambda$

Therefore

$Var(X) = E[X^2] - (\overbrace{E[X]}^{\lambda})^2$

$= \lambda^2 + \lambda - \lambda^2$

$= \lambda$