Triangular distribution

If $Z = X + Y$, $X \perp\!\!\!\perp Y$

$\mathbb{1}_{[0,1]}(a-y) \rightarrow \mathbb{1}_{[0,1]}(y)$

$$f_Z(a) = \int_{\mathbb{R}} f_X(a-y)\, f_Y(y)\, dy$$

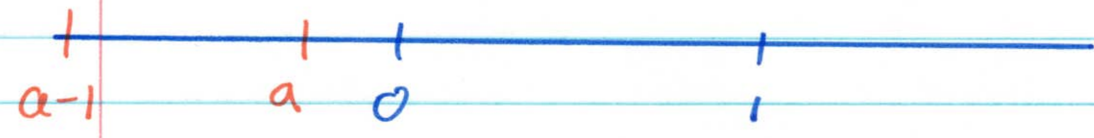
$$= \int_0^1 \mathbb{1}_{[0,1]}(a-y)\, dy \qquad 0 \leq a-y \leq 1 \Leftrightarrow a-1 \leq y \leq a$$

$$= \int_0^1 \mathbb{1}(a-1 \leq y \leq a)\, dy$$

$$= \text{Length}\left([a-1, a] \cap [0,1] \right)$$

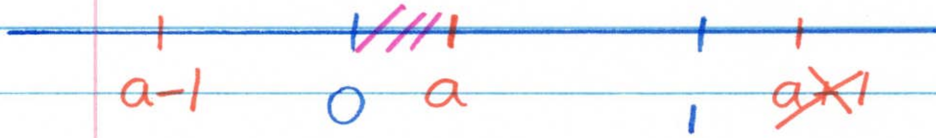
$$= \left| [a-1,a] \cap [0,1] \right|$$

Case $a < 0$



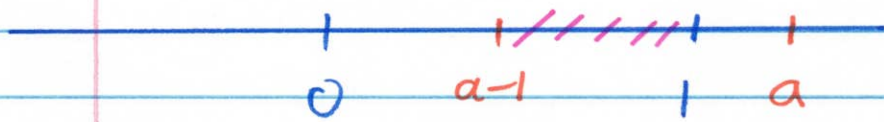
$$\left| [a-1, a] \cap [0,1] \right| = 0$$

Case $0 \le a < 1$



$$|[a-1, a] \cap [0,1]| = a$$

Case $1 \le a < 2$



$$|[a-1, a] \cap [0,1]| = 1-(a-1) = 2-a$$

Case $a \ge 2$ $\rightsquigarrow$ check

$$|[a-1, a] \cap [0,1]| = 0$$

Summary: $|[a-1,a] \cap [0,1]|$

$$= a\, \mathbb{1}_{[0,1)} + 2-a\, \mathbb{1}_{[1,2)} = f_Z(a)$$

Basketball

Model Set

X_A = # ~~gam~~ wins against class A teams

X_B = # wins " " B "

X_A = # successes in a Bernoulli trial with $n_A = \cancel{44}$ 26 experiments and $p = .4$

We get $X_A \sim Bin(\underset{26}{n_A}, \underset{.4}{p_A})$

$X_B \sim Bin(\underset{18}{n_B}, \underset{.7}{p_B})$

De Moivre

(n_A large, p_A not small)

$X_A \approx N(\mu_A, \sigma_A^2)$

where $\mu_A = n_A p_A = 10.24$

$\sigma_A^2 = n_A p_A (1-p_A) = 6.24$

$X_B \approx N(\mu_B, \sigma_B^2)$ $\mu_B = 12.60$ $\sigma_B^2 = 3.78$

Basketball (2) . ① Set

$$X^+ = X_A + X_B$$

Then $\quad X_A \sim \mathcal{N}(\mu_A, \sigma_A^2)$

$\quad\quad\quad X_B \sim \mathcal{N}(\mu_B, \sigma_B^2)$

$\quad\quad\quad X_A \perp\!\!\!\perp X_B$

Thus $\quad X^+ \sim \mathcal{N}(\underbrace{\mu_A + \mu_B}_{\simeq 23}, \underbrace{\sigma_A^2 + \sigma_B^2}_{\simeq 10.2})$

We wish to compute

$$\mathbb{P}(X^+ \geq 25) = \mathbb{P}(X^+ \geq 24.5)$$

$$= \mathbb{P}\left(\frac{X^+ - 23}{\sqrt{10.2}} \geq \frac{24.5 - 23}{\sqrt{10.2}} \right)$$

De Moivre $\quad \sim \mathcal{N}(0,1)$

$$\simeq \mathbb{P}(Z \geq .4739)$$

$$= 1 - \Phi(.4739)$$

$$= .3178$$

Basketball (3). Set

② $X^- = X_A - X_B$

Then

$$X^- \approx \mathcal{N}\left(\overbrace{\mu_A - \mu_B}^{-2.2}, \overbrace{\sigma_A^2 + \sigma_B^2}^{10.2}\right)$$

We wish to compute

$$\mathbb{P}(X^- > 0)$$

$$= \mathbb{P}(X^- \geq 0.5)$$

$$= \mathbb{P}\left(\frac{X^- + 2.2}{\sqrt{10.2}} \geq \frac{.5 + 2.2}{\sqrt{10.2}}\right)$$

DeMoivre

$$\simeq \mathbb{P}\left(\overset{\mathcal{N}(0,1)}{Z} \geq .8530\right)$$

$$= 1 - \phi(.8530)$$

$$= .1968$$

Sum of Poisson.

$X_1 \sim \mathcal{P}(\lambda_1)$ $\quad X_2 \sim \mathcal{P}(\lambda_2)$

$X_1 \perp\!\!\!\perp X_2$ $\quad Z = X_1 + X_2$.

Let $n \geq 0$. Then

$\mathbb{P}(Z = n) = ?$

If $Z = X_1 + X_2 = n$, possibilities:

$X_1 = 0$, $X_2 = n$

$X_1 = 1$, $X_2 = n-1$

$\vdots$

$X_1 = n$, $X_2 = 0$

Thus

$$\mathbb{P}(X_1 + X_2 = n) = \mathbb{P}\left(\bigcup_{k=0}^{n} \overbrace{(X_1 = k, X_2 = n-k)}^{\text{disjoint events}}\right)$$

$$= \sum_{k=0}^{n} \mathbb{P}(X_1 = k, X_2 = n-k)$$

Sum of Poisson (2)

We have obtained

$$\mathbb{P}(X_1+X_2=n) = \sum_{k=0}^{n} \mathbb{P}(X_1=k, X_2=n-k)$$

$X_1 \perp\!\!\!\perp X_2$

$$= \sum_{k=0}^{n} \mathbb{P}(X_1=k)\,\mathbb{P}(X_2=n-k)$$

$$= \sum_{k=0}^{n} e^{-\lambda_1}\frac{\lambda_1^k}{k!} \; e^{-\lambda_2}\frac{\lambda_2^{n-k}}{(n-k)!}$$

$\binom{n}{k}$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{n!} \sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!}\,\lambda_1^k\,\lambda_2^{n-k}$$

Binomial

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{n!} (\lambda_1+\lambda_2)^n$$

↳ pmf of $\mathcal{P}(\lambda_1+\lambda_2)$

Thus $X_1+X_2 \sim \mathcal{P}(\lambda_1+\lambda_2)$