Example of # successes

Set, for $i = 1, \dots, n$

A_i = " success for the i-th experiment "

$$X_i = \mathbb{1}_{A_i} = \begin{cases} 1 & \text{if } A_i \text{ occurs} \\ 0 & \text{otherwise} \end{cases}$$

Thus

$$X_i \sim \mathcal{B}(p_i) \implies E[X_i] = p_i$$

$$X = \sum_{i=1}^{n} X_i$$

Therefore

$$E[X] = E\left[\sum_{i=1}^{n} X_i\right]$$

$$\overset{\text{Prop 16}}{=} \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} p_i$$

Particular situation:

- A_i's are $\perp\!\!\!\perp$
- $P(A_i) = p$ (constant prob. of success)

Then $X = \sum_{i=1}^{n} X_i \sim Bin(n, p)$

According to the previous formula,

$$E[X] = \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} p$$

$$= np$$

We recover the formula for $E[X]$ when $X \sim Bin(n,p)$

Basic facts about F

$F(x) = P(X \leq x)$

$F(x) - F(y) = P(y < X \leq x)$

$1 - F(x) = P(X > x)$

$F(x^-) = P(X < x)$

$\hookrightarrow$ points to the left of x

On our example

$F(1) = \frac{2}{3}$

$F(1^-) = \frac{1}{2}$

Information which can be read on the cdf F

$$P(X<3) = F(3^-) = \frac{11}{12}$$

$$P(X=1) = P(X \leq 1) - P(X<1)$$
$$= F(1) - F(1^-)$$
$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

$$P(X > \tfrac{1}{2}) = 1 - F(\tfrac{1}{2})$$
$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(2 < X \leq 4)$$
$$= F(4) - F(2)$$
$$= 1 - \frac{11}{12} = \frac{1}{12}$$