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**MATH 416 PROBABILITY SPRING 2019  
PROFESSOR WANG**

**Exam 1 (5 problems, maximum 50 points)  
Monday, February 11, 2019**

You have 60 minutes to complete this exam. The exam is closed-book, closed-note.

Unless otherwise stated, show all of your work; heroic simplification is unnecessary. Full credit may not be given for an answer alone, and partial credit may be given for facts relevant to the solution. Multiple answers for any problem earn **zero** credit.

Do not unstaple pages. **Loose pages will be ignored.**

Good luck!

1. (a) (5 pts) How many solutions are there to the linear equation  $x_1 + \cdots + x_{10} = 20$ , where  $x_i \geq 1$  for all  $i = 1, \dots, 10$ ? (You don't need to compute the final number)

(b) (3 pts) How many solutions are there to the linear equation  $x_1 + \cdots + x_{10} \leq 9$ , where  $x_i \geq 1$  for all  $i = 1, \dots, 10$ ? (Give the final answer)

(c) (2 pts) Suppose that the events  $A$ ,  $B$ , and  $C$  have the following properties:

$$\mathbb{P}(A) = 0.6 \quad \mathbb{P}(B) = 0.5 \quad \mathbb{P}(C) = 0.2 \quad \mathbb{P}(B|A) = 0.4$$

$A$  and  $C$  are disjoint,  $B$  and  $C$  are independent. Then what is  $\mathbb{P}((A \cup B) \cap C)$ ?

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2. Two coins are in a hat. The coins look alike, but one coin is fair (with probability  $1/2$  of Heads), while the other coin is biased, with probability  $1/4$  of Heads. One of the coins is randomly pulled from the hat, without knowing which of the two it is. Call the chosen coin "Coin C". Coin C is tossed twice.

(a) (5 pts) What is the probability that both tosses show Heads ? (Compute the final answer)

(b)(5 pts) Given that both tosses show Heads, what is the probability that Coin C is the fair coin? (Compute the final answer)

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3. (5pt) A bridge hand of 13 cards is randomly dealt from a standard deck of 52 cards. What is the probability that the hand is void in (i.e., is missing) at least one suit? (Do not need to compute)

4. (10 pts) Let  $X$  be a random variable with cumulative distribution function  $F$  given by

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{4}, & 0 \leq x < 1, \\ \frac{1}{3}, & 1 \leq x < 2, \\ \frac{x+6}{12}, & 2 \leq x < 3, \\ 1, & 3 \leq x. \end{cases}$$

Compute

(a) (3 pts)  $\mathbb{P}(X = 1)$ ,

(b) (3 pts)  $\mathbb{P}(3 < X \leq 7)$ ,

(b) (2 pts)  $\mathbb{P}(1 \leq X \leq 2)$ ,

(d) (2 pts)  $\mathbb{P}(X \geq 3/2)$ .

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5. Consider  $n$  rolls of a fair die. Let the random variable  $X$  be the number of times that the top face is NOT 6.

(a) (2 pts) Find possible values of  $X$ . (3 pts) Find the PMF of  $X$ .

(b) (5 pts) Find the expectation of  $X$ .

(c) (5 pts) If the variance of  $X$  is  $\sigma^2$ , what is the second moment of  $X$ ? (in terms of  $\sigma$ )

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Total \_\_\_\_\_

Problem 1 \_\_\_\_\_

Problem 2 \_\_\_\_\_

Problem 3 \_\_\_\_\_

Problem 4 \_\_\_\_\_

Problem 5 \_\_\_\_\_

**Additional scratch paper**

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