Continuous random variables

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Mostly taken from *A first course in probability* by S. Ross

So far: Discrete r.v.

$$X \in \mathcal{E} = \{x_i \,;\, i \geq 1\}$$

However many important situations are modeled by $X \in \mathbb{R}$ → continuous r.v.

Translation from discrete to continuous

pmf p → density f

$\sum$ → $\int$

Outline

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General definition

Discrete case, if $B \subset \{x_i; i \geq 1\}$,
$$\mathbb{P}(X \in B) = \sum_{x_i \in B} p(x_i)$$

Definition 1.

Let

- $\mathbf{P}$ a probability on a sample space S
- $X : S \to \mathcal{E}$ a random variable, with $\mathcal{E} \subset \mathbb{R}$

measure theory

We say that X is a continuous random variable if
$\hookrightarrow$ There exists $f \geq 0$ such that for "all" $B \subset \mathbb{R}$ we have

$$\mathbf{P}(X \in B) = \int_B f(x)\, dx$$

The function f is called
$\hookrightarrow$ the probability density function of the random variable X

Law of X according to f

Type of information obtained with f: We have

$$\mathbf{P}(a \leq X \leq b) = \int_a^b f(x)\,dx$$

$$\mathbf{P}(X = a) = 0 = \int_a^a f(x)dx$$

$$F(a) = \mathbf{P}(X \leq a) = \int_{-\infty}^a f(x)\,dx$$

$$= \mathbf{P}(X \in (-\infty, a])$$

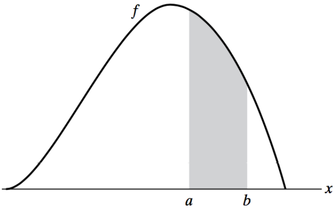

Figure: $\mathbf{P}(a \leq X \leq b) = \int_a^b f(x)\,dx$

Example: radio tube (1)

Situation:

- X = lifetime of a radio tube
- Density of X:

$$f(x) = \frac{100}{x^2} \mathbf{1}_{(100,\infty)}(x)$$

- We have 5 tubes in a set

Question: Probability that 2 of the 5 tubes have to be replaced within the first 150h of operation

Let $X \sim f(x)\,dx$

with $f(x) = \dfrac{100}{x^2} \mathbb{1}_{(100,\infty)}(x)$

<u>Rmk</u>

$$\boxed{\mathbb{P}(X < 100)} = \int_{-\infty}^{100} f(x)\,dx$$

$$= \int_{-\infty}^{100} \frac{100}{x^2} \underbrace{\mathbb{1}_{(100,\infty)}(x)}_{=0 \text{ on } (-\infty,100]}\,dx$$

$$\boxed{= 0}$$

Here we assume that the tube function for at least 100 h

Step 1 We wish to compute

$\boxed{\mathbb{P}(X \leq 150)}$

$f(x) = \frac{100}{x^2} \mathbb{1}_{(100,\infty)}(x)$

$$= \int_{-\infty}^{150} f(x)\,dx = \int_{100}^{150} \frac{100}{x^2}\,dx$$

$$= 100 \int_{100}^{150} \frac{dx}{x^2}$$

$\int \frac{dx}{x^2} = -\frac{1}{x}$

$$= 100 \left. -\frac{1}{x} \right|_{100}^{150}$$

$$= 100 \left(\frac{1}{100} - \frac{1}{150} \right) = 1 - \frac{10}{15} \boxed{= \frac{1}{3}}$$