# Outline

# Defining generating functions

**Definition 1.**

Let

- $a = \{a_i;\ i \geq 0\}$ sequence
- $s \in \mathbb{R}$

Then the generating function of $a$ is

$$G_a(s) = \sum_{i=0}^{\infty} a_i s^i \,,$$

provided the series converges

## Example of gen. fct

**Sequence** :

$$a_n = (\cos(\theta) + i\sin(\theta))^n = e^{in\theta}, \quad n \geq 0$$

**Gen fct**

$$G_a(s) = \sum_{n=0}^{\infty} a_n s^n = \sum_{n=0}^{\infty} e^{in\theta} s^n = \sum_{n=0}^{\infty} (e^{i\theta} s)^n$$

converges if $|s| < 1$

$$G_a(s) = \frac{1}{1 - s e^{i\theta}}$$

# De Moivre's series

Sequence: We consider $\theta \in [0, 2\pi]$ and

$$a_n = e^{\imath n\theta} = [\cos(\theta) + \imath \sin(\theta)]^n$$

Generating function: Defined by

$$G_a(s) = \sum_{n=0}^{\infty} a_n s^n = \sum_{n=0}^{\infty} e^{\imath n\theta} s^n$$

Computation of the generating function: For $|s| < 1$ we get

$$G_a(s) = \frac{1}{1 - s e^{\imath\theta}}$$