Neural Networks, NIP and Definable Valuations

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Abstract

In 1971, the related notions *VC dimension* and *"not the indepence property" (NIP)* were established independently of each other. The former was introduced by Vapnik and Chervonenkis [6] in the context of linear learning theory, the latter by Shelah [5] in the study of stable theories – a highly abstract concept from model theory. It took 21 years until a connection between VC dimensions and NIP was noticed in [4], describing how the two notions give different descriptions of the same idea. This connection resulted in an interesting application of a purely model theoretical and combinatorial concept to neural network learning.

In my talk, I will firstly outline the mathematical idea behind artificial neural networks and in this regard describe the formal learning process for such a network. I will then highlight the theorem which links neural network learning to the model theoretic concept of NIP theories. Finally, I will present our recent progress in the study of NIP ordered fields and definable valuations motivated by the Shelah–Hasson Conjecture (cf. [1, 2, 3]).

The talk should be accessible to anyone with a background in mathematics, and all relevant notions will be introduced.

References

[1] L. S. KRAPP, 'Algebraic and Model Theoretic Properties of O-Minimal Exponential Fields', doctoral thesis, Universität Konstanz, 2019.

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[4] M. C. LASKOWSKI, 'Vapnik–Chervonenkis classes of definable sets', *J. London Math. Soc. (2)* 45 (1992) 377–384.

[5] S. SHELAH, 'Stability, the f.c.p., and superstability; model theoretic properties of formulas in first order theory', *Ann. Math. Logic* 3 (1971) 271–362.

[6] V. N. VAPNIK and A. YA. CHERVONENKIS, 'The uniform convergence of frequencies of the appearance of events to their probabilities', *Teor. Verojatnost. i Primenen.* 16 (1971) 264–279 (Russian), *Theor. Probability Appl.* 16 (1971) 264–280 (English).