INTEGER-VALUED DEFINABLE FUNCTIONS: FROM PÓLYA TO WILKIE

Abstract

The interaction between model theory and diophantine geometry began with a new proof, by Pila and Zannier, of the Manin-Mumford conjecture. And this led to a breakthrough with Pila's proof of the André-Oort conjecture for products of modular curves. In a related direction, Jones, Thomas and Wilkie[2012] applied improvements of the Pila-Wilkie theorem for certain curves to prove results on integer-valued functions, that is, functions f such that $f(n)$ is an integer for integer points in the domain of f definable in the real exponential field. This gives a version of a 100 year old theorem due to Pólya, but with complex functions replaced by real functions.

Theorem 1 (Pólya)**.** *If $f : \mathbb{C} \to \mathbb{C}$ is entire with $f(\mathbb{N}) \in \mathbb{Z}$ and $|f(z)| \leqslant dC^{|z|}$ with real d and $C < 2$, then f is a polynomial.*

More recently, Wilkie[2016] proved an almost exact analogue of Pólya's theorem in $\mathbb{R}_{an,\exp}$. This talk will show how to combine Wilkie's ideas with techniques from transcendental number theory and o-minimality in order to establish Pólya-type theorems in which the function is definable in o-minimal expansion of real ordered field and only assumed to be integer valued on a certain sequence of natural numbers. If time permits, I will also introduce some results about several variables and then mention further research that can be done in the future.