A EXAMPLE OF PROOF

Let A be an $m \times n$-matrix with a row consisting entirely of zeros. Show that if B is $n \times p$-matrix, then AB has a row of zeros.

Proof. Write $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{n\times p}$. Assume that the entries of l-th row of A are all zeros. We claim that the l-th row of AB is a row of zeros. To see this, pick an entry c_{lj} in l-th row of AB. By the definition of multiplication of AB, we have

$$c_{lj} = a_{l1}b_{1j} + a_{l2}b_{2j} + \cdots + a_{ln}b_{nj} = \sum_{k=1}^{n} a_{lk}b_{kj}.$$

Since the l-th row of A is zero, we have $a_{l1} = a_{l2} = \cdots = a_{ln} = 0$. Hence $c_{lj} = 0$. So the l-th row of AB is a row of zeros. $\square$