A ball with mass 2 kg is thrown upward with initial velocity 100 m/s from the ground. Assume the air resistance is 0.2|v|. For simplicity, just assume that \( g = 10 \).

(1) Find the velocity \( v(t) \) when the ball goes up.

\[ m \frac{dv}{dt} = -mg - 0.2|v| \]

with initial value \( v(0) = 100 \). Since \( v > 0 \) when the ball goes up, we have \( |v| = -v \). So we get \( v' = -g - 0.2v/m \). Consider the formula for equation \( y' = ay - b \) with \( y(0) = y_0 \) is

\[ y = \frac{b}{a} + (y_0 - \frac{b}{a})e^{at}. \]

We get \( a = -0.1 \) and \( b = 10 \) here. Then

\[ v(t) = -100 + (100 + 100)e^{-0.1t} = -100 + 200e^{-0.1t}. \]

(2) Find the maximal height that ball reaches.

\[ \text{Solutions:} \] Let \( t_0 \) be the time that ball stops to arise. Hence \( v(t_0) = 0 \). That is, \( 0 = -100 + 200e^{-0.1t} \). We solve \( t_0 = -10 \ln (1/2) = 10 \ln 2 \). The maximal height the distance that ball travel at time \( t = t_0 \). Then

\[ x(t_0) = \int_0^{t_0} (-100t + 200e^{-0.1t})dt = -100t_0 + 2000(1 - e^{-0.1t_0}) = 1000(1 - \ln 2). \]

(3) Find the velocity \( v(t) \) when the ball goes down.

\[ \text{Solutions:} \] Since the air resistance is upwards, we have \( mv' = -mg + 0.2|v| \). But \( v \) is always negative and then \( |v| = -v \). So we still get the equation \( mv' = -mg - 0.2v \). Hence we get the same equation as the before. So we get \( v(t) = -100 + 200e^{-0.1t} \).

If you start time \( t = 0 \) for the time the ball start to fall. We get the answer

\[ v(t) = -100 + 200e^{-0.1(t+t_0)}. \]