QUIZ 2

A ball with mass 2 kg is thrown upward with initial velocity 100 m/s from the ground. Assume the air resistance is $0.2|v|$. For simplicity, just assume that $g = 10$.

(1) Find the velocity $v(t)$ when the ball goes up.

Solutions: Set positive direction upwards. We have

$$m\frac{dv}{dt} = -mg - 0.2|v|$$

with initial value $v(0) = 100$. Since $v > 0$ when the ball goes up, we have $|v| = -v$. So we get $v' = -g - 0.2v/m$. Consider the formula for equation $y' = ay - b$ with $y(0) = y_0$ is

$$y = \frac{b}{a} + (y_0 - \frac{b}{a})e^{at}.$$

We get $a = -0.1$ and $b = 10$ here. Then

$$v(t) = -100 + (100 + 100)e^{-0.1t} = -100 + 200e^{-0.1t}.$$

(2) Find the maximal height that ball reaches.

Solutions: Let t_0 be the time that ball stops to arise. Hence $v(t_0) = 0$. That is, $0 = -100 + 200e^{-0.1t}$. We solve $t_0 = -10\ln(1/2) = 10\ln 2$. The maximal height the distance that ball travel at time $t = t_0$. Then

$$x(t_0) = \int_0^{t_0} (-100t+200e^{-0.1t})dt = -100t_0+2000(1-e^{-0.1t_0}) = 1000(1-\ln 2).$$

(3) Find the velocity $v(t)$ when the ball goes down.

Solutions: Since the air resistance is upwards, we have $mv' = -mg + 0.2|v|$. But v is always negative and then $|v| = -v$. So we still get the equation $mv' = -mg - 0.2v$. Hence we get the same equation as the before. So we get $v(t) = -100+200e^{-0.1t}$. If you start time $t = 0$ for the time the ball start to fall. We get the answer

$$v(t) = -100 + 200e^{-0.1(t+t_0)}.$$