EXTRA CREDIT QUESTION 1

What are other good ideas to solve the initial value problem?

$$y' = f(t, y), \quad y(a) = y_0$$

We want to know (approximate) $y(b) =?$ for $b > a$.

The following is one of my idea which you can try:

Show that at least theoretically we can use Taylor expansion

$$f(y) = \sum_{n=0}^{\infty} \frac{f^{(n)}(t_0)}{n!}(b-a)^n$$

to approximate $y(b)$.

Hint: Compute $f^{(n)}(t_0)$ from the equation $y' = f(t, y)$.