

Use singular value decomposition of  $A$  to solve the following question:

- (1) Show that  $A^*A$  and  $AA^*$  share the same eigenvalues  $\lambda_i$ , and if  $\lambda_i \neq 0$  then  $\lambda_i = \sigma_i^2$  where  $\sigma_i$  is singular value of  $A$ .
- (2) Suppose that  $A$  is normal with eigenvalues  $\lambda_i$ . Let  $\sigma_i$  be singular values of  $A$ . Show that if  $\lambda_i \neq 0$  then  $\sigma_i = |\lambda_i|$ .
- (3) Find an example to show that the above statement is false without the assumption that  $A$  is normal.